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ON THE MAGNETIC SHIELDING EFFECT OF TRI-LAMELLAR SPHERICAL AND CYLINDRICAL SHELLS.

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THE subject of magnetic shielding has already received considerable attention, and, at the present day with the spread of the trolley system and the consequent increase of disturbing earth currents, the subject often demands attention from those having to do with suspended magnetic systems of any sort.

For a long time it was the custom to use extremely thick single iron shells as a protecting device. A single shell is in a measure effective, but in most cases one-tenth of the shielding material used could, if properly distributed, be made to produce many times the shielding actually produced by the usual distribution.

The advantage to be gained by using concentric shells separated by unmagnetic inter-spaces has been shown by Professor Rücker and by Professor du Bois. The former has given a general solution of the problem of multi-lamellar shielding with spherical shells; the results of this mathematical paper[1] are somewhat difficult to apply in actual shielding practice partly because there is involved the necessity of constant interpretation of the not very explicit equations used in the spherical harmonic analysis. Furthermore the practical advantages of cylindrical shells are much greater than those of spherical shells.

[1] Rücker, Phil. Mag. (5), 37, p. 95, 1894.

(Convenient cylindrical shells may be made by rolling up thin sheet iron strips of proper width.) Professor du Bois [1] has treated the case of bi-lamellar shielding both for spherical and cylindrical shells, and has, moreover, applied the results of his theoretical and experimental investigations in the construction of multi-lamellar ironclad galvanometers.[2] Experience showed that in cases of considerable disturbance it was desirable to use three shields instead of two as a protecting device.

Acting upon the suggestion of Professor du Bois the present writer has deduced the general explicit formulæ giving the shielding effect of tri-lamellar spherical and cylindrical shells under conditions to be mentioned later and in such mathematical form as to be readily interpreted. These general formulæ have then been applied in the graphical discussion of a particular problem well adapted to show the great advantage to be gained by introducing suitable air gaps within the shielding material.

In virtue of the formal analogy in the treatment of the spherical and cylindrical problems the corresponding equations for the two cases are given together at each step in the development.

We shall suppose a uniform magnetic field of strength H_e to be impressed upon the system of shells. We shall deal with the case where the space *within* the shells is to be shielded against H_e. Further we shall suppose the axes of the cylinders to be perpendicular to the lines of force; that the spherical and cylindrical shells are concentrically arranged; that the permeability is constant and equal for all shells. This last assumption involves, of course, a restriction in the values of H_e to those not exceeding, say .01 C.G.S. units, producing small variations in the magnetic condition of the shells superimposed upon their intrinsic magnetization which may be zero or have a finite residual value. It is true that this limitation invalidates the discussion for strong fields; but when we remember that the trouble with galvanometers arises from disturbing fields of the order of magnitude mentioned we may rest assured that our formulæ are capable of practical application in the designing of shielding apparatus.

[1] du Bois, Wied. Ann., 63, p. 348, 1897, and 65, p. 1, 1898. The Electrician, Vol. 40, 1898.

[2] du Bois and Rubens, Verhandl. physik. Gesellsch. Berlin, 17, p. 100, 1898.

Now, suppose a uniform disturbing field H_e to be impressed upon our tri-lamellar shielding systems, Fig. 1 representing both the spherical and cylindrical systems. The letters r_1, R_1, r_2, R_2, r_3, R_3,

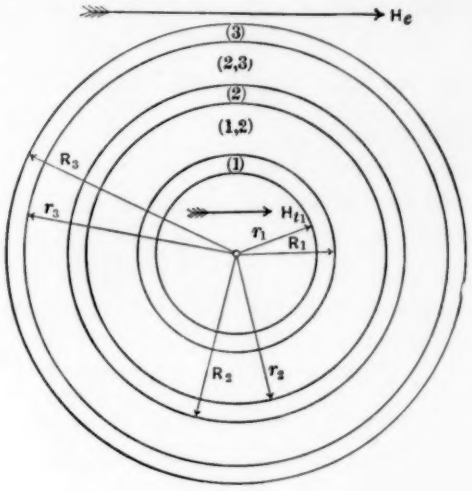

Fig. 1.

represent the inner and outer radii of the various shells; the small letters always referring to the inner radii and the large to the outer. We shall find a notation similar to that introduced by Professor du Bois in his treatment of the bi-lamellar problem to be convenient. It is given below in tabular form:

Spheres.		Cylinders.	
$p_1 = \frac{r_1^3}{R_1^3}$	$m_1 = 1 - \frac{r_1^3}{R_1^3}$	$q_1 = \frac{r_1^2}{R_1^2}$	$n_1 = 1 - \frac{r_1^2}{R_1^2}$
$p_2 = \frac{r_2^3}{R_2^3}$	$m_2 = 1 - \frac{r_2^3}{R_2^3}$	$q_2 = \frac{r_2^2}{R_2^2}$	$n_2 = 1 - \frac{r_2^2}{R_2^2}$
$p_3 = \frac{r_3^3}{R_3^3}$	$m_3 = 1 - \frac{r_3^3}{R_3^3}$	$q_3 = \frac{r_3^2}{R_3^2}$	$n_3 = 1 - \frac{r_3^2}{R_3^2}$
$p_{12} = \frac{R_1^3}{r_2^3}$	$m_{12} = 1 - \frac{R_1^3}{r_2^3}$	$q_{12} = \frac{R_1^2}{r_2^2}$	$n_{12} = 1 - \frac{R_1^2}{r_2^2}$
$p_{23} = \frac{R_2^3}{r_3^3}$	$m_{23} = 1 - \frac{R_2^3}{r_3^3}$	$q_{23} = \frac{R_2^2}{r_3^2}$	$n_{23} = 1 - \frac{R_2^2}{r_3^2}$

We shall consider the cylinders as infinitely long; but experience has shown that the equations to be derived hold in the central

portion of cylinders whose length is not less than three or four times the radius.

Now suppose the total resultant field in the direction of H_e within the innermost shell to be denoted by H_{in}. The ratio of H_e to H_{in} may be taken as a measure of the effectiveness of the shielding device. Denote this ratio by g then $g = \frac{H_e}{H_{in}}$. $g = \infty$ would, of course, mean perfect shielding.

Now it becomes necessary to find an equation expressing the relation existing among the various quantities entering into the problem, namely, "the shielding ratio" g, the permeability μ and the geometrical data given by the p's and m's in the case of the spherical shells, the q's and n's in the case of the cylindrical shells.

FUNDAMENTAL EQUATIONS.

Since Fig. 1 represents a plan section of our double system of shells, the axis of the cylindrical system will be perpendicular to the plane of the paper. The direction of the impressed field is indicated by the arrows. There will be produced upon each of the six surfaces of separation between iron and air surface distributions of "free magnetism." In the theory of such distributions it is shown that a distribution upon a spherical or cylindrical surface produces within the surface, when the impressed force is uniform, a uniform magnetic field which is coincident with the impressed field but acting in the opposite direction and of different intensity. If S denote any one of the surfaces of the spherical system and H_e the field produced within S by the distribution upon S, the theory shows that at a point without S, P say, the radial force produced by the distribution upon S is $-2\frac{r^3}{R^3}H_e \cos\vartheta$, where r is the radius of S and R the distance from the center of S to the point P and the tangential force at P (perpendicular to the radial force) is $-\frac{r^3}{R^3}H_e \sin\vartheta$, where ϑ is the angle between the radius to P and the impressed field. With cylindrical shells, for any surface S and a point P without S we should have the corresponding forces $-\frac{r^2}{R^2}H_e \cos\vartheta$, and

$-\frac{r^2}{R^2} H_e \sin \vartheta$. Now considering the surface S we may think of the resultant force within it to be made up of two parts one of which is due to causes external and the other to causes internal. Call the first H_e and the second H_i. Starting in each case with the inner surface we may write down the following equations, applying to both spherical and cylindrical systems.

$$R \begin{cases} H_{i1} = H_{i2} + H_{i1} = H_e + H_{i6} + H_{i5} + H_{i4} + H_{i3} + H_{i2} + H_{i1} \\ H_{i2} = H_{i3} + H_{i2} = H_e + H_{i6} + H_{i5} + H_{i4} + H_{i3} + H_{i2} \\ H_{i3} = H_{i4} + H_{i3} = H_e + H_{i6} + H_{i5} + H_{i4} + H_{i3} \\ H_{i4} = H_{i5} + H_{i4} = H_e + H_{i6} + H_{i5} + H_{i4} \\ H_{i5} = H_{i6} + H_{i5} = H_e + H_{i6} + H_{i5} \\ H_{i6} = H_{i7} + H_{i6} = H_e + H_{i6} \end{cases}$$

From what has been said above and from Fig 1 the meaning of the notation is clear.

We do not require to know the exact distribution of the lines of force throughout the shells but the quantities H_{i6}, H_{i5}, H_{i4}, H_{i3}, H_{i2}, H_{i1} must be such as to satisfy the two surface conditions namely: that at surfaces of separation of iron from air the tangential components of the magnetic force must be equal in the two media and that the normal component of the magnetic induction must be continuous. The first condition, from what has been already said, may be seen to be always satisfied whatever the values of $H_{i6} \cdots H_{i1}$. But in order that the condition of continuity in the normal component of the magnetic induction be satisfied we must have the following equations also satisfied. In these equations μ represents the permeability of the iron. The equations pertaining to the spherical shells are given first. In both sets of equations cos θ occurs as a common factor in both members of each equation but is not written down.

$$
\left.
\begin{array}{l}
H_{i1} = \mu(H_{i2} - 2H_{i1}) \\
\mu\left(H_{i2} - 2\dfrac{r_1^3}{R_1^3}H_{i1}\right) = H_{i3} - 2H_{i2} - 2\dfrac{r_1^3}{R_1^3}H_{i1} \\
H_{i3} - 2\dfrac{R_1^3}{r_3^3}H_{i2} - 2\dfrac{r_1^3}{r_2^3}H_{i1} = \mu\left(H_{i4} - 2H_{i3} - 2\dfrac{R_1^3}{r_2^3}H_{i2} - 2\dfrac{r_1^3}{r_2^3}H_{i1}\right) \\
\mu\left(H_{i4} - 2\dfrac{r_2^3}{R_2^3}H_{i3} - 2\dfrac{R_1^3}{R_2^3}H_{i2} - 2\dfrac{r_1^3}{R_2^3}H_{i1}\right) = H_{i5} - 2H_{i4} - 2\dfrac{r_2^3}{R_2^3}H_{i3} - 2\dfrac{R_1^3}{R_2^3}H_{i2} - 2\dfrac{r_1^3}{R_2^3}H_{i1} \\
H_{i5} - 2\dfrac{R_2^3}{r_3^3}H_{i4} - 2\dfrac{r_2^3}{r_3^3}H_{i3} - 2\dfrac{R_1^3}{r_3^3}H_{i2} - 2\dfrac{r_1^3}{r_3^3}H_{i1} = \mu\left(H_{i6} - 2H_{i5} - 2\dfrac{R_2^3}{r_3^3}H_{i4} - 2\dfrac{r_2^3}{r_3^3}H_{i3} - 2\dfrac{R_1^3}{r_3^3}H_{i2} - 2\dfrac{r_1^3}{r_3^3}H_{i1}\right) \\
\mu\left(H_{i6} - 2\dfrac{r_3^3}{R_3^3}H_{i5} - 2\dfrac{R_2^3}{R_3^3}H_{i4} - 2\dfrac{r_2^3}{R_3^3}H_{i3} - 2\dfrac{R_1^3}{R_3^3}H_{i2} - 2\dfrac{r_1^3}{R_3^3}H_{i1}\right) \\
= H_{i6} - 2H_{i6} - 2\dfrac{r_3^3}{R_3^3}H_{i5} - 2\dfrac{R_2^3}{R_3^3}H_{i4} - 2\dfrac{r_2^3}{R_3^3}H_{i3} - 2\dfrac{R_1^3}{R_3^3}H_{i2} - 2\dfrac{r_1^3}{R_3^3}H_{i1}
\end{array}
\right\} S
$$

The corresponding set of equations for the cylinders is

$$
\left.
\begin{aligned}
H_{i1} &= \mu(H_{i2} - H_{i1}) \\
\mu\left(H_{i2} - \frac{r_1^2}{R_1^2}H_{i1}\right) &= H_{i3} - H_{i2} - \frac{r_1^2}{R_1^2}H_{i1} \\
H_{i3} - \frac{R_1^2}{r_2^2}H_{i2} - \frac{r_1^2}{r_2^2}H_{i1} = \mu\left(H_{i4} - H_{i3} - \frac{R_1^2}{r_2^2}H_{i2} - \frac{r_1^2}{r_2^2}H_{i1}\right) \\
\mu\left(H_{i4} - \frac{r_2^2}{R_2^2}H_{i3} - \frac{R_1^2}{R_2^2}H_{i2} - \frac{r_1^2}{R_2^2}H_{i1}\right) &= H_{i5} - H_{i4} - \frac{r_2^2}{R_2^2}H_{i3} - \frac{R_1^2}{R_2^2}H_{i2} - \frac{r_1^2}{R_2^2}H_{i1} \\
H_{i5} - \frac{R_2^2}{r_3^2}H_{i4} - \frac{r_2^2}{r_3^2}H_{i3} - \frac{R_1^2}{r_3^2}H_{i2} - \frac{r_1^2}{r_3^2}H_{i1} = \mu\left(H_{i6} - H_{i5} - \frac{R_2^2}{r_3^2}H_{i4} - \frac{r_2^2}{r_3^2}H_{i3} - \frac{R_1^2}{r_3^2}H_{i2} - \frac{r_1^2}{r_3^2}H_{i1}\right) \\
\mu\left(H_{i6} - \frac{r_3^2}{R_3^2}H_{i5} - \frac{R_2^2}{R_3^2}H_{i4} - \frac{r_2^2}{R_3^2}H_{i3} - \frac{R_1^2}{R_3^2}H_{i2} - \frac{r_1^2}{R_3^2}H_{i1}\right) \\
= H_e - H_{i6} - \frac{r_3^2}{R_3^2}H_{i5} - \frac{R_2^2}{R_3^2}H_{i4} - \frac{r_2^2}{R_3^2}H_{i3} - \frac{R_1^2}{R_3^2}H_{i2} - \frac{r_1^2}{R_3^2}H_{i1}
\end{aligned}
\right\} T
$$

Treatment of Fundamental Equations.

From the set of equations (S) we may eliminate the H_i's with the aid of the set (R) giving for the spherical shells after combination and rearrangement of terms the following set:

$$
\begin{aligned}
&(\mu-1)H_{i6}+(\mu-1)H_{i5}+(\mu-1)H_{i4}+(\mu-1)H_{i3}+(\mu-1)H_{i2}\\
&\qquad-(2\mu+1)H_{i1}\equiv-(\mu-1)H_{e}\\
&(\mu-1)H_{i6}+(\mu-1)H_{i5}+(\mu-1)H_{i4}+(\mu-1)H_{i3}+(\mu+2)H_{i2}\\
&\qquad-2\frac{r_1^3}{R_1^3}(\mu-1)H_{i1}\equiv-(\mu-1)H_{e}\\
&(\mu-1)H_{i6}+(\mu-1)H_{i5}+(\mu-1)H_{i4}-(2\mu+1)H_{i3}-\frac{2R_1^3}{r_2^3}(\mu-1)H_{i2}\\
&\qquad-2\frac{r_1^3}{r_2^3}(\mu-1)H_{i1}\equiv-(\mu-1)H_{e}\\
&(\mu-1)H_{i6}+(\mu-1)H_{i5}+(\mu+2)H_{i4}-2\frac{r_2^3}{R_2^3}(\mu-1)H_{i3}-2\frac{R_1^3}{R_2^3}(\mu-1)H_{i2}\\
&\qquad-2\frac{r_1^3}{R_2^3}(\mu-1)H_{i1}\equiv-(\mu-1)H_{e}\\
&(\mu-1)H_{i6}-(2\mu+1)H_{i5}-2\frac{R_2^3}{r_3^3}(\mu-1)H_{i4}-2\frac{r_2^3}{r_3^3}(\mu-1)H_{i3}-2\frac{R_1^3}{r_3^3}(\mu-1)H_{i2}\\
&\qquad-2\frac{r_1^3}{r_3^3}(\mu-1)H_{i1}\equiv-(\mu-1)H_{e}\\
&(\mu+2)H_{i6}-2\frac{r_3^3}{R_3^3}(\mu-1)H_{i5}-2\frac{R_2^3}{R_3^3}(\mu-1)H_{i4}-2\frac{r_2^3}{R_3^3}(\mu-1)H_{i3}-2\frac{R_1^3}{R_3^3}(\mu-1)H_{i2}\\
&\qquad-2\frac{r_1^3}{R_3^3}(\mu-1)H_{i1}\equiv-(\mu-1)H_{e}.
\end{aligned}
$$

The corresponding set for the cylinders is

$$
\begin{array}{cccccc}
(\mu-1)H_{16}+ & (\mu-1)H_{25}+ & (\mu-1)H_{14}+ & (\mu-1)H_{23}+ & (\mu-1)H_{12}- & (\mu+1)H_{11}=-(\mu-1)H_{1'} \\
(\mu-1)H_{16}+ & (\mu-1)H_{25}+ & (\mu-1)H_{14}+ & (\mu-1)H_{23}+ & (\mu+1)H_{12}-\frac{r_1^2}{R_1^2}(\mu-1)H_{11}=-(\mu-1)H_{1'} \\
(\mu-1)H_{16}+ & (\mu-1)H_{25}+ & (\mu-1)H_{14}- & (\mu+1)H_{13}-\frac{R_1^2}{r_2^2}(\mu-1)H_{12}-\frac{r_1^2}{r_2^2}(\mu-1)H_{11}=-(\mu-1)H_{1'} \\
(\mu-1)H_{16}+ & (\mu-1)H_{25}+ & (\mu+1)H_{14}-\frac{r_2^2}{R_2^2}(\mu-1)H_{13}-\frac{R_1^2}{R_2^2}(\mu-1)H_{12}-\frac{r_1^2}{R_2^2}(\mu-1)H_{11}=-(\mu-1)H_{1'} \\
(\mu-1)H_{16}- & (\mu+1)H_{25}-\frac{R_2^2}{r_3^2}(\mu-1)H_{14}-\frac{r_2^2}{r_3^2}(\mu-1)H_{13}-\frac{R_1^2}{r_3^2}(\mu-1)H_{12}-\frac{r_1^2}{r_3^2}(\mu-1)H_{11}=-(\mu-1)H_{1'} \\
(\mu+1)H_{16}-\frac{r_3^2}{R_3^2}(\mu-1)H_{15}-\frac{R_2^2}{R_3^2}(\mu-1)H_{14}-\frac{r_2^2}{R_3^2}(\mu-1)H_{13}-\frac{R_1^2}{R_3^2}(\mu-1)H_{12}-\frac{r_1^2}{R_3^2}(\mu-1)H_{11}=-(\mu-1)H_{1'}
\end{array}
$$

We have now two sets of six linear equations in the six quantities $H_{i1} \cdots H_{i6}$, one set corresponding to the spherical shells and the other to the cylindrical shells. We have sufficient equations in each case to enable us to evaluate $H_{i1} \cdots H_{i6}$ in terms of the geometrical data, the permeability and the impressed field.

It will be convenient to treat these equations by means of determinants, and the method used in treating the spherical problem only will be given; the cylindrical problem is subject to a treatment mathematically analogous to that indicated for the spherical problem.

Referring to the set of equations applying to spherical shells, let the determinant formed by taking in order the coefficients of the H's in the left-hand member of the first equation for the elements of its first row, the coefficients, in order, of the H's in the left-hand member of the second equation for the elements of its second row and so on be called Δ. Let the determinant which is equal to Δ except in the first column, where in place of the elements found in the first column is put in the place of each element the common right-hand member of the system of equations namely—$(\mu - 1)H_e$, be called $\Delta 1$. Let $\Delta 2$, $\Delta 3$, $\Delta 4$, $\Delta 5$ and $\Delta 6$ be defined in an analogous way. The determinants thus defined are seen to be of the sixth order.

We shall have then according to Leibnitz's rule for the solution of simultaneous linear equations the following values of $H_{i1} \cdots H_{i6}$ pertaining to the spherical shells

$$H_{i1} = \frac{\Delta 1}{\Delta}$$
$$\vdots \quad \vdots \quad \vdots$$
$$H_{i6} = \frac{\Delta 6}{\Delta}.$$

These are the values to be substituted in the equation

$$H_{t1} = H_e + H_{i6} + \cdots + H_{i1}$$

and it will be remembered H_{t1} stands for the field within the innermost spherical shell and H_e for the impressed field. From the method of formation of $\Delta 1 \cdots \Delta 6$ it will be seen that each of these determinants contains H_e as a factor, and the shielding ratio $g = \frac{H_e}{H_{t1}}$

is found from the equation just given. Substituting in this equation the values found for $H_{i1} \cdots H_{i6}$ we get

$$H_{i1} = H_e \left\{ 1 + \frac{1}{H_e}\left[\frac{\Delta 1 + \Delta 2 \cdots \Delta 6}{\Delta}\right] \right\}$$

and therefore

$$g = \frac{H_e}{H_{i1}} = \frac{He\Delta.}{\Delta H_e + \Delta 1 + \Delta 2 + \cdots + \Delta 6}.$$

This then expressed symbolically in determinantal form is the result sought. It remains now to expand the Δ's and reduce the somewhat complicated resultant expression. It will be sufficient to illustrate the method used in expanding the various determinants to consider one of them only, say Δ.

It is in the ordinary notation

$$\begin{vmatrix}
(\mu-1), & (\mu-1), & (\mu-1), & (\mu-1), & (\mu-1), & -(2\mu+1) \\
(\mu-1), & (\mu-1), & (\mu-1), & (\mu-1), & (\mu+2), & -2(\mu-1)\frac{r_1^3}{R_1^3} \\
(\mu-1), & (\mu-1), & (\mu-1), & -(2\mu+1), & -2(\mu-1)\frac{R_1^3}{r_2^3}, & -2(\mu-1)\frac{r_1^3}{r_2^3} \\
(\mu-1), & (\mu-1), & (\mu+2), & -2(\mu-1)\frac{r_2^3}{R_2^3}, & -2(\mu-1)\frac{R_1^3}{R_2^3}, & -2(\mu-1)\frac{r_1^3}{R_2^3} \\
(\mu-1), & -(2\mu+1), & -2(\mu-1)\frac{R_2^3}{r_3^3}, & -2(\mu-1)\frac{r_2^3}{r_3^3}, & -2(\mu-1)\frac{R_1^3}{r_3^3}, & -2(\mu-1)\frac{r_1^3}{r_3^3} \\
(\mu+2), & -2(\mu-1)\frac{r_3^3}{R_3^3}, & -2(\mu-1)\frac{R_2^3}{R_3^3}, & -2(\mu-1)\frac{r_2^3}{R_3^3}, & -2(\mu-1)\frac{R_1^3}{R_3^3}, & -2(\mu-1)\frac{r_1^3}{R_3^3}
\end{vmatrix}$$

Multiply the sixth row by R_3^3, the fifth by r_3^3, the fourth by R_2^3, the third by r_2^3, the second by R_1^3, the first by r_1^3. To keep the value of the determinant unchanged we must multiply the resulting form by $\frac{1}{R_3^3 \cdots r_1^3}$. Now subtract the fifth row from the sixth, the fourth from the fifth and so on down to the first row. These operations have not changed the value of the determinant. Now in the resultant form subtract the second column from the first, the third from the second and so on up to the sixth. Divide the sixth row of the resultant expression by R_3^3, the fifth by r_3^3 and so on down to the

first row. To keep the value of the determinant unchanged multiply the resulting form by $R_3^3 \ldots r_1^3$. Employing the p and m notation explained on page 195 we may write the resulting form of the determinant Δ as follows :

$$\Delta = \begin{vmatrix} 0 & 0 & 0 & 0 & 3\mu & -(2\mu+1) \\ 0 & 0 & 0 & -3 & (\mu+2)m_1 & 3p_1 \\ 0 & 0 & 3\mu & -(2\mu+1)m_{12} & -3\mu p_{12} & 0 \\ 0 & -3 & (\mu+2)m_2 & 3p_2 & 0 & 0 \\ 3\mu & -(2\mu+1)m_{23} & -3\mu p_{23} & 0 & 0 & 0 \\ (\mu+2)m_3 & 3p_3 & 0 & 0 & 0 & 0 \end{vmatrix}$$

As the determinant now stands Laplace's Method of development will be found convenient. By this method the determinant Δ is at once seen to be equal an expression with two terms, each of which is the product of two determinants of the third order. Using the "criss-cross" method applicable to determinants of the third order the expansion of these determinants is easily obtained and thus the expansion of Δ. In a similar way the expansions of $\Delta 1 \ldots \Delta 6$ may be obtained.

We have now to substitute the values found for Δ, $\Delta 1 \ldots \Delta 6$ in the expression

$$g = \frac{\Delta . H_e}{\Delta H_e + \Delta 1 + \Delta 2 + \ldots + \Delta 6}.$$

Assuming this to have been done and the proper simplifications and rearrangement of the terms to have been accomplished we get

$$\begin{aligned} g - 1 = {} & \tfrac{2}{9} \frac{(\mu-1)^2}{\mu} \Big\{ (1 - p_1 p_2 p_3) \\ & + \tfrac{1}{81} \frac{(2\mu+1)^2 (\mu+2)^2}{\mu^2} m_1 m_{12} m_2 m_{23} m_3 \\ & + \tfrac{1}{9} \frac{(2\mu+1)(\mu+2)}{\mu} [(m_1 m_3 + m_1 m_2 - m_1 m_2 m_3) m_{12} \\ & + (m_1 m_3 + m_2 m_3 - m_1 m_2 m_3) m_{23} - m_1 m_3 m_{12} m_{23}] \Big\}. \end{aligned} \tag{A}$$

A similar treatment of the fundamental equations for the cylindrical shells gives the corresponding formula

$$g - 1 = \tfrac{1}{4}\frac{(\mu-1)^2}{\mu}\Big\{(1 - q_1q_2q_3) + \tfrac{1}{16}\frac{(\mu+1)^4}{\mu^2}\, n_1n_{12}n_2n_{23}n_3$$

$$(B) \qquad + \tfrac{1}{4}\frac{(\mu+1)^2}{\mu}\left[(n_1n_3 + n_1n_2 - n_1n_2n_3)\, n_{12}\right.$$

$$\left. + (n_1n_3 + n_2n_3 - n_1n_2n_3)n_{23} - n_1n_3n_{12}n_{23}\right]\Big\}$$

Equations (A) and (B) are capable of transformation into the following forms:

$$g = \left\{\left(\tfrac{2}{9}\frac{(\mu-1)^2}{\mu}m_1 + 1\right)\left(\tfrac{2}{9}\frac{(\mu-1)^2}{\mu}m_2+1\right)\left(\tfrac{2}{9}\frac{(\mu-1)^2}{\mu}m_3+1\right)\right\}$$

$$+ \tfrac{2}{9}\frac{(\mu-1)^2}{\mu}\cdot\frac{(2\mu+1)(\mu+2)}{9\mu}\left\{\frac{(2\mu+1)(\mu+2)}{9\mu}m_1m_2m_3p_{12}p_{23}\right.$$

(A')

$$- \tfrac{2}{9}\frac{(\mu-1)^2}{\mu}(m_1m_2m_3p_{12} + m_1m_2m_3p_{23}) - (m_1m_2p_{12} + m_2m_3p_{23}$$

$$\left. + m_1m_3p_{12}p_{23})\right\}.$$

$$g = \left\{\left(\tfrac{1}{4}\frac{(\mu-1)^2}{\mu}n_1+1\right)\left(\tfrac{1}{4}\frac{(\mu-1)^2}{\mu}n_2+1\right)\left(\tfrac{1}{4}\frac{(\mu-1)^2}{\mu}n_3+1\right)\right\}$$

(B')

$$+ \tfrac{1}{16}\frac{(\mu^2-1)^2}{\mu^2}\left\{\frac{(\mu+1)^2}{4\mu}n_1n_2n_3q_{12}q_{23}\right.$$

$$\left. - \tfrac{1}{4}\frac{(\mu-1)^2}{\mu}(n_1n_2n_3q_{12}+n_1n_2n_3q_{23}) - (n_1n_2q_{12}+n_2n_3q_{23}+n_1n_3q_{12}q_{23})\right\}.$$

A few experimental determinations of the shielding ratio of three cylindrical shells made of transformer iron have furnished a satisfactory verification of the formula (B').

Discussion of the Equations.

Consider first equations (A) and (B). g is seen to be a function of the geometrical quantities represented by the p's and the m's, the q's and the n's and of the permeability μ. In both (A) and (B) the expression for g is of the third degree in μ. In both g equals unity when

$\mu = 1$ and ∞ when $\mu = \infty$. [These remarks apply also to equations (A') and (B').] If any one of the three shells be allowed to vanish we have the case of two shells and the form of the resulting equations is seen to be independent of the shell which we choose to make vanish. Suppose in the case of three spherical shells we make the outer one vanish by causing the outer radius to become equal to the inner radius. Equation (A) then takes the form

$$g - 1 = \tfrac{2}{9} \frac{(\mu - 1)^2}{\mu} \left\{ (1 - p_1 p_2) + \frac{(2\mu + 1)(\mu + 2)}{9\mu} m_1 m_2 m_{12} \right\}.$$

This is identically the equation given for two spherical shells by Professor du Bois[1] and a similar reduction for the cylindrical case would give the corresponding formula for two cylindrical shells. If we suppose the two outer spherical shells to vanish we obtain the well-known formula expressing the shielding effect of a single shell

$$g - 1 = \tfrac{2}{9} \frac{(\mu - 1)^2}{\mu} (1 - p_1).$$

Referring to the equations (A), (B), (A'), (B'), it is to be noticed that the subscripts 1 and 3 may be throughout interchanged without changing the equations; that is to say the formulæ are symmetrical with respect to the inner and outer shells.

Of course (A') and (B') are capable of discussion in a manner similar to that given above for (A) and (B), difference of form in the results occurring. The forms (A') and (B') are particularly adapted to a form of the discussion to be given now.

Suppose the outer shell to have so far expanded that there is no "magnetic interference" between it and the middle shell ($p_{23} = 0$, $m_{23} = 1$) and so far as to be also consistent with the condition that the middle shell may so expand as to cause the mutual interference between it and the inner shell to vanish ($p_{12} = 0$, $m_{12} = 1$) and yet the mutual interference between the middle and outer shell to remain zero. In this assumed expansion the ratio of the radii of each shell is supposed to remain constant. This means, of course, that R_3, r_3, R_2, r_2, become very large. The field within the outer shell may be now considered to act directly upon the middle

[1] du Bois, Wied. Ann., 63, p. 353. 1898.

shell as an impressed field; denote it by H_1. Likewise the field within the middle shell may be considered to act directly upon the inner shell as an impressed field; denote it by H_2. Denote the field within the inner shell by H_1. We have now

$$g = \frac{H_e}{H_3} \cdot \frac{H_3}{H_2} \cdot \frac{H_2}{H_1} = g_3 g_2 g_1$$

and have so expressed what Professor du Bois designates as a multiplication of shielding ratios, g_3, g_2, g_1 representing the shielding effect of the outer, middle and inner shell respectively against the field impressed upon each of them. We have then

$$g_3 = \frac{H_e}{H_3} = \tfrac{2}{9} \frac{(\mu - 1)^2}{\mu} m_3 + 1,$$

$$g_2 = \frac{H_3}{H_2} = \tfrac{2}{9} \frac{(\mu - 1)^2}{\mu} m_2 + 1,$$

$$g_1 = \frac{H_2}{H_1} = \tfrac{2}{9} \frac{(\mu - 1)^2}{\mu} m_1 + 1,$$

for it will be remembered that the form of the expression giving the shielding effect of a single spherical shell is $g = \frac{2}{9} \frac{(\mu - 1)^2}{\mu} m + 1$. Now in the expansion of the shells mentioned above it was specified that the ratio of the inner to the outer radius should remain constant for each shell. This means p_1, p_2 and p_3 and therefore m_1, m_2 and m_3 must also remain constant. So we may write equations (A') and (B') as follows:

$$(A'') \quad g = g_1 g_2 g_3 + \tfrac{2}{9} \frac{(\mu - 1)^2}{\mu} \frac{(2\mu + 1)(\mu + 2)}{9\mu} \left\{ \frac{(2\mu + 1)(\mu + 2)}{9\mu} \right.$$

$$m_1 m_2 m_3 p_{12} p_{23} - \tfrac{2}{9} \frac{(\mu - 1)^2}{\mu} (m_1 m_2 m_3 p_{12} + m_1 m_2 m_3 p_{23})$$

$$\left. - (m_1 m_2 p_{12} + m_2 m_3 p_{23} + m_1 m_3 p_{12} p_{23}) \right\}$$

$$(B'') \quad g = g_1 g_2 g_3 + \tfrac{1}{16} \frac{(\mu^2 - 1)^2}{\mu^2} \left\{ \frac{(\mu + 1)^2}{4\mu} n_1 n_2 n_3 q_{12} q_{23} \right.$$

$$- \tfrac{1}{4} \frac{(\mu - 1)^2}{\mu} (n_1 n_2 n_3 q_{12} + n_1 n_2 n_3 q_{23}) - (n_1 n_2 q_{12} + n_2 n_3 q_{23}$$

$$\left. + n_2 n_3 q_{12} q_{23}) \right\}$$

(B'') being derived for the cylindrical shells in a manner analogous to that used for (A''). A consideration of (A'') and (B'') shows that the combined effect of the terms following $g_1g_2g_3$, is strongly subtractive, that is the shielding is much greater when the shells are very far removed from one another and as we have seen the shielding ratio in this case is equal to $g_1g_2g_3$. We may think then of the product $g_1g_2g_3$ as representing an "ideal" shielding and the terms following this product as representing the departure from this ideal case caused by the shells mutually interfering.

If μ is large, say > 100, (A) and (B) may be simplified as follows

$$(A_1)\quad \begin{aligned} g - 1 = {} & \tfrac{2}{9}(\mu-2)\{(1-p_1p_2p_3) + \tfrac{4}{81}(\mu+1)(\mu \\ & + 2)m_1m_{12}m_2m_{23}m_3 + \tfrac{2}{9}(\mu+2)[(m_1m_3 + m_1m_2 \\ & - m_1m_2m_3)m_{12} + (m_1m_3 + m_2m_3 - m_1m_2m_3)m_{23} \\ & - m_1m_3m_{12}m_{23}]\} \end{aligned}$$

$$(B_1)\quad \begin{aligned} g - 1 = {} & \tfrac{1}{4}(\mu-2)\{(1-q_1q_2q_3) + \tfrac{1}{16}(\mu+2)^2 n_1n_{12}n_2n_{23}n_3 \\ & + \tfrac{1}{4}(\mu+2)[(n_1n_3 + n_1n_2 - n_1n_2n_3)n_{12} + (n_1n_3 + n_2n_3 \\ & - n_1n_2n_3)n_{23} - n_1n_3n_{12}n_{23}]\}. \end{aligned}$$

Similar approximate formulæ might be deduced from the equations (A'), (B'), (A''), (B'') and often prove useful.

GRAPHICAL DISCUSSION OF A PARTICULAR PROBLEM.

Professor Rücker has discussed analytically the question of maximum shielding, with reference to the weight of material used, for several particular problems. Among other things he has shown that if the radius a of a spherical shielded space be fixed and the amount of shielding material given the latter is best employed in the form of a *single* shell until a limit t is reached defined by $\frac{t}{a} = \frac{3}{2\mu}$ (approximately) where t denotes the thickness. So for a comparatively small shielded space, since μ for the best material reaches the initial value 300, it is only for extremely thin shells that the shielding with a single shell is, strictly speaking, most economical as regards weight. He has shown further that with two and three

spherical shells when the permeability is great the arrangement with regard to minimum weight of material is best when the radii of the successive bounding surfaces are in geometrical progression, the ratio of the innermost to the outermost radius being supposed given. Professor Rücker's discussion is limited to spherical shells.

Professor du Bois has discussed the bi-lamellar case for both spherical and cylindrical shells when the innermost radius is given, and has found the best arrangement to be that for which the radial ratio of the air-gap is 1.5538 for cylindrical shells and 1.3815 for spherical shells; strictly speaking these results only hold for two thin similar shells (*i. e.*, of equal radial ratios); but the best radial ratios of the air space, as calculated for other less simple cases in actual practice, never come out much different from the above values; hence for designing purposes it may be roughly fixed between 1.3 and 1.4 for spheres, and between 1.5 and 1.6 for cylinders.

The following graphical discussion is introduced in the hope of showing in a manner convincing and easy to understand the advantages of lamination of the shielding material and not with the intent to give the most general possible discussion of the equations used in plotting the curves. At the same time it is hoped that the general explicit equations given above and the following illustration of their applicability to the plotting of shielding curves may be of use to those who may have occasion to design shielding apparatus.

We shall discuss here the shielding of single, bi-lamellar and trilamellar spherical and cylindrical shells under the supposition that the permeability of the shielding material is, in all cases, 202, which is but about two-thirds of that of the best material hithero tested, and under other conditions to be mentioned later.

Referring to Fig. 2, the curves shown relate to single spherical and cylindrical shells. The abscissæ represent the radial ratios of the shells; if the radius of the shielded space should be made equal to unity ($r = 1$) then the abscissæ would represent the successive values of the outer radius of the shell. The ordinates of curves 1 and 2 represent the shielding ratio, g. Curves 1 and 2 show how the shielding increases as the thickness of the shells increases. At first the gradient of the cylindrical curve is not quite so sharp as that of the spherical; later as the thickness increases it becomes

greater and does not reach its asymptotic value so soon as in the spherical curve. Curve 1 reaches very nearly its asymptotic value when its outer radius is say three times the inner. The corresponding curve 2 for the hollow cylinder has not quite reached this point even when the outer radius has become five times the inner. These

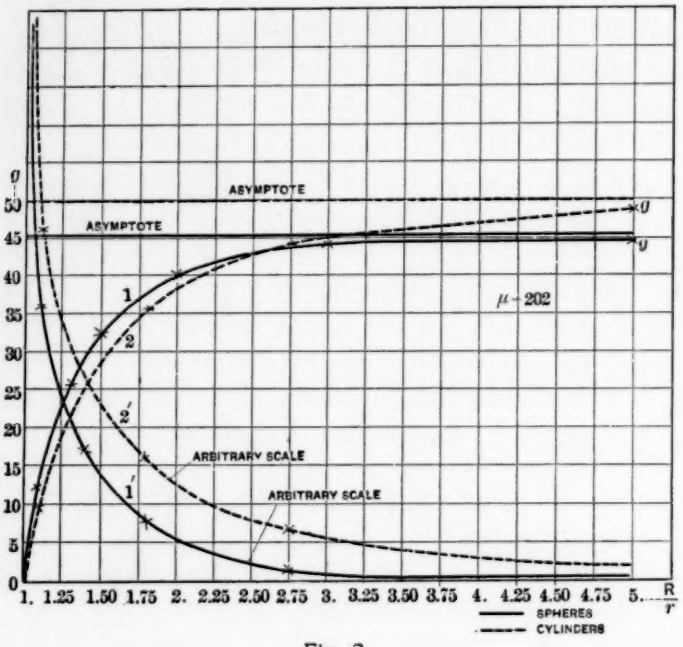

Fig. 2.

curves show clearly the futility of increasing the thickness of a single shielding shell beyond a certain point. Curves 1′ and 2′, corresponding to spherical and cylindrical shells respectively, represent, upon a scale quite arbitrary in each case as regards the ordinates, the way in which the shielding per unit weight of material varies as the thickness of the shells is increased.

Consider now Fig. 3. It refers to the case of two concentric spherical shells, also to that of two concentric cylindrical shells. If r and R denote the innermost and outermost radii respectively of either system of shells, a condition imposed is that $\frac{R}{r}$ shall always equal five. The other conditions are that the two shells must be

equal in thickness and that $\mu = 202$. If the radius of the shielded space be taken equal to unity ($r = 1$) then $R = 5$. The figures given as the abscissæ in the present case and in the case of three shells are to be considered on the supposition that $r = 1$. The independent variable is the thickness of the air-gap. It may vary between the limits 0 and 4. The upper of the two rows of abscissæ represents the varying air-gap thickness and the lower the corresponding thickness of the shells. The ordinates represent the shielding ratio g. Curves 1 and 2 are without reference to the weight and refer to spherical and cylindrical shell respectively. Curves 1′ and 2′ represent upon scales quite arbitrary as regards the ordinates the way in which the shielding per unit weight varies as the air-gap is increased or decreased.

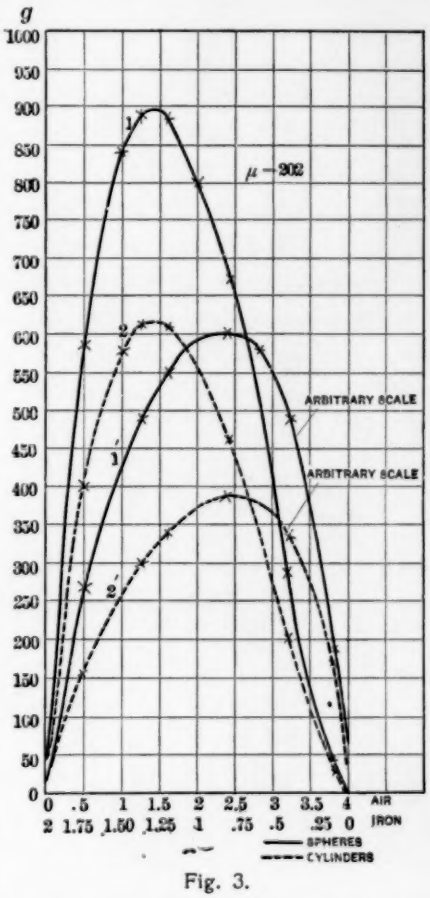

Fig. 3.

While graphically it is difficult to specify the exact position of the maximum points, yet it may be seen that the maxima for curves 1 and 2 occur for very nearly the same value of the air-gap distance and for that value for which the thickness of the shells and the airgap is the same. But the maxima for curves 1′ and 2′ occur much further to the right when the shells have become notably thinner; and approximately for the same value of the air-gap thickness and for that value which makes the air-gap thickness about three times the common thickness of the shells. By means of two shells arranged as specified above it would be possible, in the case of

spherical shells, to reduce our impressed field to about $\frac{1}{900}$ of its original value within the space to be shielded. Had all of the available shielding space been filled up with iron we should have obtained but about $\frac{1}{20}$ as complete shielding and should have used about 1.3 as much iron, showing clearly the advantage of using air instead of iron in some parts of the shielding arrangement, and also how the addition of iron at the wrong place may be decidedly harmful.

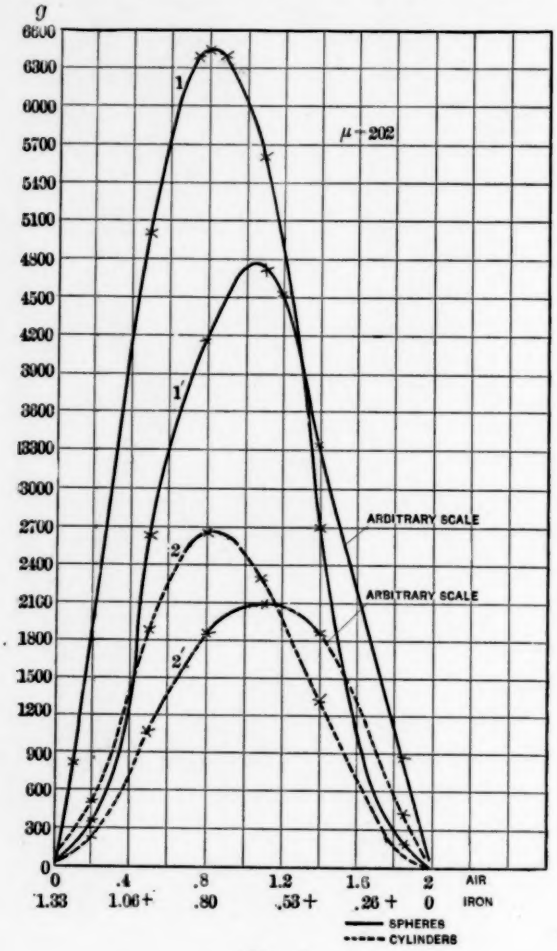

Fig. 4.

In Fig. 4 we have represented the case of three spherical shells and also of three cylindrical shells under conditions quite similar to

those laid down for the bi-lamellar shells. The conditions are: that $\mu = 202$; that the shells have always a common thickness; that the two air-gaps have the same thickness; that $r = 1$, $R = 5$, r and R denoting the innermost and outermost radius respectively of the shells in both cases (spherical and cylindrical). Curves 1 and 2 refer, without reference to the weight, to the spherical and cylindrical shells respectively. Curves 1′ and 2′ are the corresponding curves representing the variation in the shielding per unit weight. The upper of the two rows of abscissæ represents the common thickness of the two air-gaps and it is the thickness which has been taken as the independent variable. The second row represents the corresponding common thickness of the three shells in both cases (spherical and cylindrical). Curves 1 and 2 reach a maximum very nearly simultaneously and for that value of the air-gap thickness at which very approximately, at any rate, the common thickness of the air-gaps equals that of the shells. Curves 1′ and 2′ whose ordinates are drawn upon independent arbitrary scales reach their maximum points nearly simultaneously and for a value of the air-gap thickness, which makes the thickness of each air-gap about twice the thickness of each shell. With three spherical shells as specified above it would be possible then to reduce an impressed field to about $\frac{1}{6400}$ of its original value within the shielded space. Had all of the available shielding space been filled up with iron we should have obtained but about $\frac{1}{160}$ as complete shielding and should have used about 1.5 as much iron. The advantages of an extended lamination of the shielding material are hereby made very obvious.

As was mentioned above, this graphic discussion is intended more as an illustrative problem than as one which would allow of the deduction of practical rules to cover the most probable conditions which might arise in practice.

In a joint paper by Professor du Bois and the writer the results of some experimental investigations with three cylindrical shells will be published.

I have to express my cordial thanks to Professor du Bois for his kind assistance during the development of the foregoing discussion.

BERLIN, April 15, 1899.

A NEW RESPIRATION CALORIMETER AND EXPERIMENTS ON THE CONSERVATION OF ENERGY IN THE HUMAN BODY, II.

W. O. ATWATER AND E. B. ROSA.

III. TESTS OF THE APPARATUS AS A CALORIMETER.

TWO series of test experiments were instituted to ascertain whether the calorimeter would accurately measure a given quantity of heat generated within the respiration chamber. In the first series an electric current of measured strength flowing through a known resistance generated in a given time a quantity of heat which could be readily calculated. As there was no circulation of air and no water vapor produced in the apparatus, it was used at the best advantage simply as a calorimeter. In the second series alcohol was burned within the chamber, a current of air circulating through it supplying the necessary oxygen and carrying away the products of combustion. Here the complications arising from a ventilating current of air and the production of a large amount of moisture were introduced. Evidently, if the calorimeter will accurately measure a known quantity of heat generated in these two ways, it may be employed with confidence to measure the heat generated by a living subject within the chamber. In the latter case, however, we can not expect as high a degree of accuracy as in the former, for the rate of evolution of heat is more variable and the movement of the person or animal about the respiration chamber introduces disturbances which will prevent the highest degree of accuracy in the measurements; and furthermore the errors due to physiological causes may be considerable.

ELECTRICAL TEST EXPERIMENTS.

The arrangement of the apparatus in the electrical tests is shown in Fig. 13. The current from the 220-volt mains passes into the

respiration chamber through a variable resistance, *RS*, a Weston ammeter, *Am*, and two copper voltameters, *Vm*, where it passes through *PQ*, a coil of German-silver wire No. 30, having a resistance of about 100 ohms. A Kelvin balance in series with a known resistance, R_1, in oil, and a Weston voltmeter in parallel with them are connected to the points *a*, *b*, and indicate the fall of potential through the resistance, *PQ*. Usually an assistant keeps the current constant by adjusting the resistance, *RS*. The current is given with considerable accuracy by the Weston ammeter, which in the first test experiment was used alone. But when the copper voltameters were employed the values for the current obtained with them were used.

Two test experiments were made on March 20 and 25, 1897, which gave very satisfactory results. A third on March 26th gave slightly more concordant results than either of the others and will be here given in some detail, in order to illustrate the operation of the calorimeter as well as to show its accuracy.

A sample page from the record of the test of March 26th, is given below. It gives the observations of one hour.

In the first column the time is recorded. In the second column are the deflections in the scale divisions in the thermo-electric circuit (No. 1) of the walls of the respiration chamber. Numbers at the left side of the column indicate deflections to the left and show that the inner chamber is a little cooler than air space *B*. At 5.10, the air in *B* having been cooled slightly, the deflection is zero. Then it becomes positive for a few minutes and the air in *B* requires slight warming. The deflections during the hour are on the average less than one division and indicate an average difference of temperature of about 0.004° C. But this difference is part of the time positive and part of the time negative, so that the algebraic average is only one-fourth of one division to the right, indicating that the inner chamber was on the average for the hour 0.0018° C. warmer than the surrounding space. By carrying these differences along from page to page the adjustments are made so that the differences cancel each other, and there is no correction to apply for flow of heat throught the walls for the whole experiment.

One division deflection corresponds, as already stated, to 0.007°

TABLE I.

Sample page from record of calorimetric observations of experiment of March 26, 1897.

Time, p. m.	Inner wall. No. 1.	Inner wall. Parts.	Water circuit. T_1, T_2.	Water circuit. T_2-T_1.	Water circuit. Bridge No. 6.	Inside temp. No. 5.	Remarks.
h. m.	−3½		°C.	°C.	°C.	°C.	
5 00	½	+2, +1, 0, −1½			12.80	22.61	
02	0				12.80	22.61	
04	0				12.79	22.61	1st 10 kilos at 5.04.00.
06	0				12.80	22.62	Av. bridge read. 12.73.
08	1				12.87	22.62	12.73−.27=12.46°.
10	0				12.85	22.62	12.46°×10=124.6
12	1½		15.97		12.84	22.61	calories.
14	1½		3.42	12.55	12.78	22.62	
16	1½				12.72	22.62	
18	1½				12.80½	22.62	
20	1	+1, +1½, −1, 0			12.81	22.62	
22	½				12.84½	22.62	
24	0				12.83½	22.62	
26	0				12.86	22.62	
28	0		15.93		12.76	22.63	
30	½		3.44		12.77	22.63	
32	0				12.87½	22.63	
34	½			12.49	12.77	22.64	
36	1				12.80½	22.64	
38	0				12.81	22.64	
40	1				12.81	22.64	
42	½		15.94		12.81	22.64	
44	½	0, −½, 0, −½	3.43	12.51	12.77	22.64	
46	0				12.78	22.65	2d 10 kilos at 5.47.45.
48	1				12.79	22.65	Av. bridge read. 12.81.
50	½				12.86	22.65	12.81−.27=12.54°.
52	0				12.83	22.65	12.54°×10=125.4
54	½				12.79	22.65	calories.
56	0		16.04		12.81	22.64	
58	0		3.47	12.57	12.85	22.64	
	7 +11½						

C. difference of temperature, and with this difference of temperature 4 small calories per minute, or 0.24 of a large calorie per hour, flow through the walls of the chamber. This was determined in the following manner: A current of 0.17 ampères was passed through a resistance within the calorimeter for four hours. The fall of potential was 43 volts, giving 7.3 watts or about 104 small

calories per minute. The temperature of the chamber rose a little at first, but was constant during the latter part of the experiment, showing that all of the heat was escaping through the walls. The deflection produced by the thermo-electric elements of circuit No. 1 was about 25 divisions, remaining substantially constant for some time before the end of the experiment. This gives 4 small calories per minute per scale division as the amount of heat flowing through the walls of the calorimeter. This may be called the radiation constant.

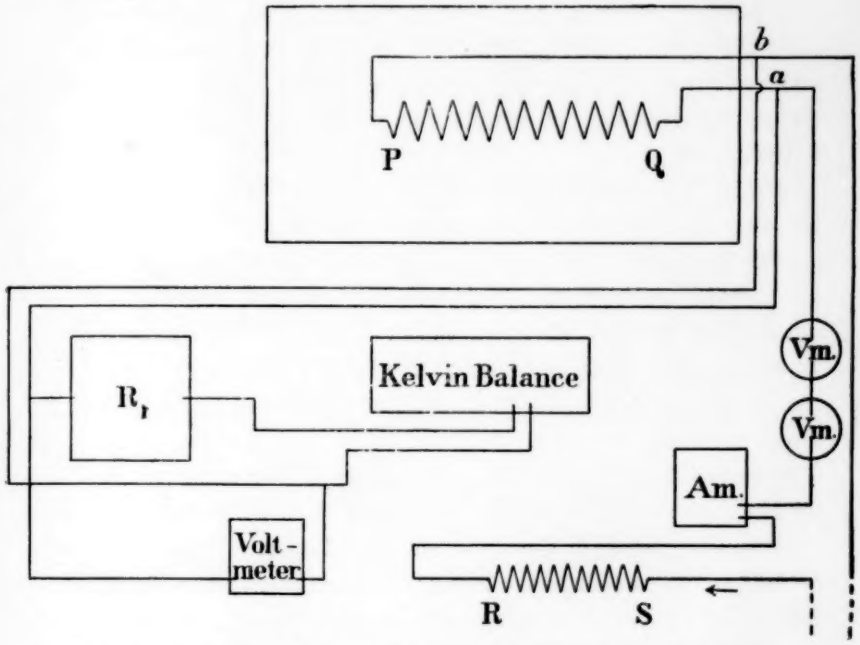

Fig. 13.—Arrangement of the electric apparatus for generating heat in the test experiments.

For an average deflection of one-fourth of one division the rate at which heat would be escaping from the interior of the calorimeter would be 0.06 calorie per hour. The amount of heat generated and carried away from the calorimeter was, in this experiment, 74 calories per hour. Thus the amount of heat passing through the walls is only $0.060 \div 74 = 0.0008$ of the total, or one-twelfth of 1 per cent., and would not be an appreciable error if

uncorrected. It is, however, subsequently recovered by keeping the deflection slightly negative, as already explained.

In the third column are recorded the deflections observed when parts of the thermo-electric circuits are separately joined to the galvanometer, the four numbers being for the top, upper, lower and bottom sections, respectively. These deflections are all small and make more certain the balance indicated by zero deflection of the entire system in series.

In the fourth column are recorded the readings of the two mercury thermometers, G and H (Fig. 6), and the fifth column shows their difference $(T_2 - T_1)$, which is the gain in temperature of the water flowing through the absorbing pipes. The sixth column shows the readings of bridge No. 6 for the same difference of temperature.

The bridge does not give the exact difference of temperature of the two coils, which are immersed in the water, its readings being always too high. However, it correctly indicates the variations of temperature between the thermometer readings, and thus interpolations between the observed thermometer readings can be made. In actual practice the average of the bridge readings is reduced by the average difference between thermometer and bridge. For example 10 kilograms ran out during the period of forty-four minutes from 5.04 to 5.48 p. m. The average of the bridge readings for the forty-four minutes, during which the 10 kilograms of water were passing through the absorbers, was 12.81. The correction to be subtracted to reduce the bridge readings to degrees centigrade is 0.27. This gives an average difference of 12.54°, and hence 125.4 calories of heat were brought away in these forty-four minutes. In the same way the difference of temperature for each 10 kilograms is determined. A tabular statement of the heat measured during the entire experiment from 4.20 p. m. to 11.37 p. m. is given in the table on next page.

If a calorie be defined as the amount of heat required to raise 1 kilogram of water 1° C., at 15° C. it will be equivalent to a definite number of ergs, which must be determined by experiment. The value given by E. H. Griffiths,[1] as the result of his elaborate work

[1] Phil. Trans. Royal Society, A, 1893. Proc. Roy. Soc., vol. 55.

TABLE II.

Summary of heat measurements, March 26, 1897.

Time.	(a) Duration of period.	Calorimetric measurements. (b) Water.	(c) Difference in temperature by bridge No. 6.	(d) Corrected difference in temperature, $c-0.27$.	(e) Amount of heat, $b \times d$.	Electrical measurements. E Voltage on coil.	I Current through coil.	W Watts, $E \times I$	(k) Calories, $W \times a \times .2378$.	Temperature of the inside by the copper thermometer. No. 5.
h. m. s.	*Secs.*	*Kilos.*	°C.	°C.	*Cals.*		*Amps.*			°C.
4 20 20	.....	....	.....	.....	.....	.....	.....	....	....	22.63
5 04 00	2,620	10	12.73	12.46	124.60	147.8	1.363	201.5	125.54	22.61
5 47 45	2,625	10	12.81	12.54	125.40	147.8	1.363	201.5	125.78	22.65
6 31 45	2,640	10	12.92	12.65	126.50	147.8	1.363	201.5	126.50	22.67
7 15 27	2,622	10	12.77	12.50	125.00	146.0	1.350	197.1	122.89	22.57
7 59 10	2,623	10	12.80	12.53	125.30	147.8	1.363	201.5	125.68	22.59
8 42 40	2,610	10	12.79	12.52	125.20	147.8	1.363	201.5	125.06	22.60
9 26 15	2,615	10	12.78	12.51	125.10	147.8	1.363	201.5	125.30	22.60
10 09 50	2,615	10	12.68	12.41	124.10	147.8	1.363	201.5	125.30	22.64
10 53 30	2,620	10	12.82	12.55	125.50	147.8	1.363	201.5	125.54	22.62
11 37 10	2,620	10	12.81	12.54	125.40	147.8	1.363	201.5	125.54	22.61
					1,252.10	Heat generated by				
				Capacity correction for 0.02°..... =	—1.2	electric current.... =			1,253.13	
				Heat measured by calorimeter..... =	1,250.9					

on the determination of the relation between the electric units and those of heat is,

$$J = 4.1982 \times 10^7 \text{ ergs, at } 15° \text{ C.}$$

The average temperature of the water in the absorbers in our experiments was in every case between 8° and 10° C. The specific heat of water[1] at 10° C. is 1.0019[2] times its value at 15° C., and hence it requires 1.0019 times as much heat to raise 1 kilogram one degree at 10° C. as at 15° C., so that

$$J = 4.2062 \times 10^7 \text{ ergs, at } 10° \text{ C.}$$

This value is practically the same as that given by Professor Rowland[3] for the mechanical equivalent of heat at 8° C., and so appears to be the most nearly correct for our use.

[1] Bartoli and Stracciati, Boll. mens. dell'Acc. Gioenia, 18, Apr. 26, 1891. Preston's Theory of Heat, p. 265.

[2] As estimated from the figures of the table on p. 231—*i. e.*, 1.0029 + 1.0010 = 1.0019.

[3] Proc. Am. Acad. Sci., 15 (1879), p. 75.

A recent recalculation of Rowland's value of the mechanical equivalent of heat[1] gives 4.200×10^7, at 8° C., and it has been suggested that the difference between this value and Griffith's is accounted for by the uncertainty in the value of the ampère, but as we use the same ampère as Griffith did we should use his value of J.

The energy of the electric current expressed in ergs being

$$CEt \times 10^7,$$

when expressed in calories becomes

$$\frac{CEt \times 10^7}{4.2062 \times 10^7} = CEt \times 0.23775.$$

Referring to the above table, it will be seen that the total amount of heat generated by the current is 1253.13 calories. The amount measured is 1252.10 calories, less than that produced by the electric current by 1.03 calories. During the experiment the temperature of the chamber fell from 22°.63 to 22°.61. The capacity for heat of the apparatus is equivalent to that of 60 kilograms of water —that is, it requires 60 calories to raise the temperature 1°. This was determined in the following manner:

The calorimeter was held at a constant temperature (23°.8) for several hours. A current of 1.77 ampères, at 33 volts, was then passed through it for two hours. The temperature slowly rose and, keeping the deflection of the No. 1 thermo-electric circuit zero, no heat was allowed to pass through the walls. At the end of this time the current was stopped and the calorimeter came to a constant temperature at 25°.4. This rise in temperature was thus 1°.6. The amount of heat generated by the electric current was 98.2 calories, or 61.5 calories per degree. Other determinations gave similar results, and the round number 60 was taken as a sufficiently exact value of the thermal capacity of the apparatus.

In the above case, where the temperature fell 0.02, there will be $.02 \times 60 = 1.2$ calories which are absorbed and measured in addition to the heat from the current. Applying this correction, we have for the measured amount of heat produced by the current $1252.10 - 1.2 = 1250.9$ calories, which is 2.2 calories, or 0.18 per cent. less than indicated by the electrical determination.

[1] Phil. Mag., 44 1897, p. 169.

The current in the above experiment was maintained constant, and the voltage measured by the Kelvin balance. About 7 p. m. the voltage on the mains was lower than usual for some minutes, and the variable portion of the resistance was insufficient to maintain the voltage on the coil. Consequently the balance was set at a lower point and the current kept constant at this smaller value until it could be restored to its former value. This makes the average voltage and current for the fourth period smaller than the others. The totals show that the amount measured was 2.2 calories, or 0.18 per cent. too small. This, however, assumes that the measurements of temperature at the beginning and the end are accurate and that the temperature of the whole apparatus is the same as that of the air. An error of a few hundredths of a degree might throw the difference in the opposite direction. Since this change is so slight, and as all the measurements of the temperature are presumably equally reliable, it would be more accurate if the average of all was used in obtaining the capacity correction. This can be done by taking different portions of the experiment and applying the capacity correction to each portion; the average of the errors from the different portions will give a more probable value of the error of the experiment. The above experiment is thus discussed in Table III.

TABLE III.

Discussion of experiment of March 26, 1897.

Periods.	(*e*) Heat as measured.	Temperature No. 5. (*f*) At the beginning.	Temperature No. 5. (*g*) At the end.	Temperature No. 5. (*h*) Change, $g-f$.	(*i*) Capacity correction, $h \times 60$.	(*j*) Corrected heat, $e+i$.	(*k*) Heat put in by the electric current.	Error in heat measurement, $j-k$.
	Cals.	°	°	°	*Cals.*	*Cals.*	*Cals.*	*Cals.*
1-6.....	752.0	22.63	22.60	−0.03	−1.8	750.2	751.5	−1.3
2-7.....	752.5	22.61	22.60	−0.01	−0.6	751.9	751.1	+0.8
3-8.....	751.2	22.65	22.64	−0.01	−0.6	750.6	750.6	0.0
4-9.....	750.2	22.67	22.62	−0.05	−3.0	747.2	749.6	−2.4
5-10.....	750.6	22.57	22.61	+0.04	+2.4	753.0	752.2	+0.8
					Sum	3,752.9	3,755.0	−2.1
					Avg.....	750.6	751.0	−0.4
	$\frac{0.4}{750.6} = 0.053$ per cent.							

In the first column is indicated the number of periods corresponding to the section of the experiment taken for discussion. In the

second, (*e*) is the heat measured during the same period—*i. e.*, the sum of the values in column (*e*), Table II. The next three columns show the change in temperature of the apparatus, and the necessary correction is given in the sixth column. Applying these corrections to the values in the second column gives the corrected heat in (*j*). The values of the heat generated, (*k*), are obtained by adding those in column (*k*), Table II. The last column gives the differences between the heat generated and that measured, and the average of these shows the error of the apparatus. In this experiment the error is about 0.05 per cent., which is as close as the electrical energy could be measured.

Another test of a somewhat different nature was made April 30th. A small incandescent electric lamp, rated at 4 candle power, but burned low, taking 4.21 watts, or 1 small calorie per second, was placed within the chamber and the heat given off was measured. The heat being so small it was necessary to have the water enter at a temperature nearly as high as the air inside, and to flow in a very slow stream. The results of six hours' run are given in Table IV.

TABLE IV.

Experiments with 4*-candle power electric lamp—Summary of heat measurements, April* 30, 1897.

Time.	(*a*) Duration of period.	Calorimetric measurements. (*b*) Water.	(*c*) Difference in temperature by bridge No. 6.	(*d*) Corrected difference in temperature. *c*—0.09°.	(*e*) Amount of heat, *b* × *d*.	Electrical measurements. *E* Voltage on lamp.	*I* Current through lamp.	*W* Watts, *E* × *I*.	(*k*) Calories, *W* × *a* × .2378.	Temperature of the inside by the copper thermometer No. 5.
h. m. s.	*Secs.*	*Kilos.*	°*C.*	°*C.*	*Cals.*		*Amp.*			°*C.*
9 56 00	.....	.....	.....	.....	.....	.....	.....	....	.....	19.72
10 11 45	945	1	1.82	1.73	1.72	5.61	0.75	4.21	0.95	19.70
10 40 35	1,730	2	1.35	1.26	2.52	5.61	.75	4.21	1.73	19.65
10 54 55	860	1	1.28	1.19	1.19	5.61	.75	4.21	.86	19.63
11 38 45	2,630	3	1.15	1.06	3.18	5.61	.75	4.21	2.63	19.62
1 55 00	8,175	9	.97	.88	7.92	5.61	.75	4.21	8.17	19.63
2 22 30	1,650	2	.41	.32	.64	5.61	.75	4.21	1.65	19.66
3 08 00	2,730	2.6	.61	.52	1.35	5.61	.75	4.21	2.73	19.72
3 54 00	2,760	3.3	.95	.86	2.84	5.61	.75	4.21	2.76	19.73
				Heat measured.....	= 21.37		Heat generated.....		= 21.48	

It will be seen that the measurement is very close. There is apparently a small capacity correction, which if applied, makes the error in the opposite direction. It is so small, however, that it is impossible to say with certainty how much the correction should be.

As this amount of heat is only about 1 per cent. of the maximum which the apparatus will measure and about 3 per cent. of the heat usually measured, it is evident that there can be no appreciable loss or gain of heat by radiation or otherwise. We therefore believe that this calorimeter, with proper manipulation, will measure heat as accurately as it is possible to measure it by any other form of calorimeter.

Alcohol Test Experiments.

The electrical tests above described demonstrate the accuracy of the apparatus as a calorimeter when the evolution of heat inside the chamber is reasonably uniform, and there is no current of air passing through and no vaporization of water within the chamber. In experiments with men, however, the development of heat is less uniform; furthermore, a current of air is passing through the chamber, water and carbon dioxid are produced, and water is vaporized within it. In these experiments the principal measurements made with the aid of the apparatus are the amounts of water and carbon dioxid produced by the man and the heat given off from his body. In experiments where the external muscular work has been considerable the latter has been transformed into electrical energy, measured, and transformed into heat. This heat has been measured with that given off directly by the body. The crucial test of the accuracy of the determinations of carbon dioxid, water and heat must be made in test experiments under circumstances closely similar to those of the actual experiments with men. When ethyl alcohol is burned in air carbon dioxid, water and heat are produced. If known quantities of alcohol be burned inside the chamber while a current of air is passing through, the conditions will approach very closely those of an experiment with man. To make these experiments accurate, the amount of alcohol burned must be exactly known and the combustion must be complete.

Several series of experiments were made in which alcohol was

burned inside the chamber. The rate of flow of the ventilating current, the method of analysis of the incoming and outgoing air, and the method of measurement of heat were the same as if a man were inside the chamber.

1. Composition of Alcohol Used—Specific Gravity.

In some of the tests here described we have used pure ethyl alcohol kindly furnished by Dr. Edward R. Squibb, but for the larger part of our work, including the alcohol tests with the respiration calorimeter, a high grade of commercial alcohol was used instead of the so-called absolute alcohol, because it can be more easily stored, weighed, and measured without change of water content. The specific gravity of this alcohol was determined at the time of each experiment. The variations in the different samples were small, the proportion of alcohol being not far from 90.5 per cent. by weight. From the percentage of alcohol present the amount of carbon dioxid and water that would be formed by complete oxidation is readily calculated as follows: Taking the atomic weight of hydrogen as 1, that of oxygen[1] is here taken as 15.88, and carbon 11.91. With these atomic weights the molecular weight of alcohol, C_2H_6O, is 45.70, of H_2O is 17.88 and of CO_2 is 43.67. The reaction in the combustion of alcohol is $C_2H_6O + O_6 = 2CO_2 + 3H_2O$. By molecular weights, $45.70 + 95.28 = 87.34 + 53.64$, or 1 gm. alcohol + 2.0848 gm. oxygen = 1.9110 gms. CO_2 + 1.1737 H_2O. Hence one gram of ethyl alcohol (C_2H_6O) will yield on oxidation 1.9110 grams of carbon dioxid and 1.1737 grams of water. One gram of a mixture of alcohol and water containing 90.77 per cent. absolute alcohol (the strength of that used in the first three tests) will give 1.9110×0.9077, or 1.7346 grams of carbon dioxid, and 1.1737×0.9077, or 1.0654 grams of water from the combustion of the ethyl alcohol in addition to the 0.0923 grams of water present in the mixture, making a total of 1.1577 grams of water from 1 gram of the alcohol mixture used. The estimates of the quantity of heat produced by the oxidation of a gram of each of the different specimens of alcohol are made by use of the factor

[1] F. W. Clarke, Journal of Am. Chem. Soc., March, 1898. Richards, Am. Chem. Journal, p. 534, July, 1898

7.067 calories for the heat of the combustion of 1 gram pure ethyl alcohol as explained beyond.

2. Heat of Combustion of Alcohol—Determination with the Bomb Calorimeter.

The heat of combustion of the alcohol used in the tests was determined by the bomb calorimeter. This well known instrument[1] which has been used in determining the heats of combustion of the food and excreta in the experiments with a human subject, is illustrated in section in Fig. 14. The so-called bomb is a steel cylinder, about 12 cm. deep and 6 cm. average interior diameter, the wall being about 0.6 cm. thick. A groove in the cover contains a lead gasket, and when the outer ring or collar is screwed down with a heavy spanner, the bomb is closed perfectly tight, even for pressures of 30 to 40 atmospheres. The cover is provided with a neck *D*. The valve screw *F* passes through a second screw *E*, and at the bottom of *E* is a lead packing *L*. The pressure of *E* upon *L* makes a perfectly tight joint about the bottom of *F*. Oxygen is admitted to the bomb through a narrow passage extending from *G* to the bottom of *F*, and then directly downward to the interior of the cylinder. This passage is closed when *F* is screwed down. The bomb is fitted with a platinum cup, spun from a single piece of platinum, which serves as lining, and the cover is also lined with platinum. This effectually prevents any oxidation of the bomb itself by the compressed oxygen during the combustion. Two platinum rods *H*, *I*, serve to hold a platinum capsule *O*, containing the substance to be burned, and also to conduct an electric current which, flowing through the fine iron wire over the capsule, fuses the wire and so ignites the substance to be burned.

The bomb is placed in a cylinder of brittania metal, 12 cm. in diameter and 22 cm. high, and about 1700 c.c. of water added, the whole being surrounded by two concentric cylinders of "indurated fiber," in the manner shown in the figure. A stirrer *SS*, is driven at a uniform speed by a small electric motor. The thermometer is

[1] See description by W. O. Atwater in Bulletin 21 of the Office of Experiment Stations of the U. S. Department of Agriculture and by W. O. Atwater and O. S. Blakeslee in report of Storrs (Conn.) Agricultural Experiment Station for 1897.

graduated to one-hundredth of a degree, and read with a magnifying glass to thousandths. Those here used were made by Fuess of Berlin and calibrated at the Physikalisch-technische Reichsan-

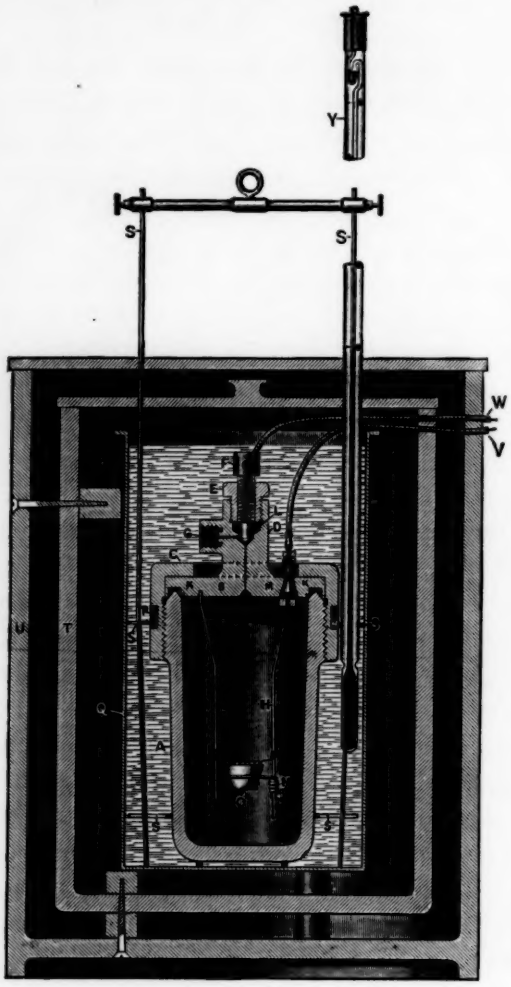

Fig. 14.

Bomb calorimeter; apparatus as used for actual determination of heats of combustion.

stalt. Oxygen is supplied by the S. S. White Dental Mfg. Co., New York in cylinders about 125 cms. long and 18 cm. in diameter, at a pressure of 200 atmospheres. It is admitted to the bomb until

the gauge indicates a pressure of 20 atmospheres. One cylinder full of oxygen will thus suffice for hundreds of combustions.

In the combustions of alcohol two methods were used. In one the alcohol was poured upon cellulose filter blocks. The weighing was made with due precautions to prevent, if possible, any loss by evaporation. The alcohol and filter block were burned in the calorimeter, a correction being applied for the heat of combustion of the filter block used. Eight determinations were made by this method with commercial alcohol of high grade and of different strengths.

The use of filter blocks is objectionable because of the difficulty of avoiding evaporation of alcohol during the process of weighing and transferring to the bomb, and because of the considerable and not absolutely certain correction to be applied for the heat of combustion of the filter blocks used. The first difficulty was avoided entirely and the other partially by the second method employed. In this the alcohol was inclosed in small gelatin capsules by which the evaporation is prevented. The weight of gelatin is also small and the correction for its heat of combustion is less than with the filter blocks. Eight determinations were made with Squibb's alcohol and nine with commercial alcohol, such as was used in the tests in the respiration calorimeter. The results obtained by both methods are shown in Table V.

The heat of combustion when determined by the first method ranged from 7,030 to 7,090, and averaged 7,069 small calories per gram of absolute alcohol.

The results obtained by the second method with Squibb's alcohol ranged from 7,044 to 7,090 small calories, and averaging 7,061 small calories per gram, absolute alcohol. The results of similar determinations with the commercial alcohol ranged from 7,045 to 7,103 small calories, and averaging 7,070 small calories per gram, absolute alcohol. The average of these 17 determinations with alcohol in gelatin capsules gives 7,066 small calories, practically the same result as was obtained by the use of the filter blocks. Considering the range of variation in these different determinations, we would hardly be justified in assuming that the figure 7,067 represents to a very high degree of accuracy the heat of combustion of

TABLE V.

Determination of heats of combustion of alcohol in the bomb calorimeter.

	Ethyl hydroxid by weight.	Heat of combustion per gram actually determined.	Calculated heat of combustion per gram of ethyl hydroxid.
Alcohol contained in filter blocks:	*Per cent.*	*Small cals.*	*Small cals.*
Specimen No. 1	89.9	6,348	7,062
Do	89.9	6,345	7,058
Do	89.9	6,377	7,093
Average of above 3	.....	.....	7,071
Specimen No. 2	81.3	5,744	7,063
Do	81.3	5,763	7,090
Average of above 2	.....	.....	7,076
Specimen No. 3	72.5	5,132	7,078
Do	72.5	5,094	7,030
Do	72 5	5,132	7,078
Average of above 3	.....	.....	7,062
Average of above 8	.....	.....	7,069
Alcohol contained in gelatin capsules:			
Specimen No. 4	88.0	6,205	7,050
Do	88.0	6,224	7,072
Do	88.0	6,209	7,055
Do	88.0	6,199	7,044
Do	88.0	6,204	7,050
Do	88.0	6,203	7,049
Do	88.0	6,227	7,076
Do	88.0	6,240	7.090
Average of above 8	.....	.....	7,061
Specimen No. 5	90.61	6,435	7,101
Do	90.61	6,384	7,046
Do	90.61	6,401	7,064
Do	90.61	6,383	7.045
Do	90.61	6,403	7,066
Do	90.61	6,397	7,060
Do	90.61	6,436	7,103
Do	90.61	6,433	7,099
Do	90.61	6,386	7,047
Average of above 9	.....	.....	7,070
Average of above 17	.....	.....	7,066
Average of above 25	.....	.....	7,067

ethyl hydroxid. There is reason to hope, however, that some of the sources of error in the determinations may yet be partially eliminated, and that thus more reliable results may be obtained. It is worthy of note that Berthelot and Matignon[1] obtained the figure 7,068 as the average of two determinations, which gave 7,067.3 and 7,068.5 small calories, respectively, at 13°. The average 7,068 corresponds to 7,079 at 20°, a value very close to those obtained here.

The heat of combustion of alcohol of any given dilution with water is found by multiplying the heat of combustion of 1 gram of absolute alcohol by the percentage present in the specimen. Thus in the specimen of alcohol used in the first three tests the heat of combustion was 7,067 × .9077, or 6,415 small calories per gram.

While the heat of combustion of 1 gram of absolute alcohol is thus 7,067 small calories per gram, this does not represent the amount of heat that is given off by the combustion of 1 gram of alcohol within the chamber of the respiration calorimeter. In the bomb calorimeter all the water vapor formed is condensed within the apparatus, and hence the heat that had been required to vaporize the water is given off again. In the respiration calorimeter, on the other hand, the water passes out as vapor in the ventilating air current. The heat required to vaporize it comes from the combustion of the alcohol, and is not measured by the calorimeter. In order to obtain the total amount of heat given off in the combustion, therefore, the heat actually measured must be added to the amount required to vaporize the excess of water in the outgoing over that in the incoming air current, which is the amount of water vaporized at the expense of the heat produced in the combustion of the alcohol.

3. The Calorie here used as the Unit of Measure.

Although the specific heat of water is often taken as unity for all temperatures, it actually varies by an appreciable amount. Hence if the unit of heat be defined as the amount required to raise unit mass of water 1 degree, this unit will be a variable one. The theoretical large calorie, namely, the quantity of heat that will raise a kilogram of water from 0° to 1°, or from 4° to 5°, is a very incon-

[1] Ann. Chim. et Phys., 6 ser., 27 (1892), p. 312.

venient unit in practice. Moreover, the specific heat of water changes quite rapidly at these low temperatures, and hence a higher temperature is more favorable for the working unit. Many authorities take the specific heat of water at 15° C. as unity. Inasmuch, however, as we have used 20° C. as our standard temperature for the respiration calorimeter in experiments with human subjects, and inasmuch also as the specific heat of water has a minimum near to 20° and changes very slowly in that vicinity, we have found it desirable to take as our working unit of heat the calorie at 20°. In other words, the large calorie at 20°, which we designate in the tables as C_{20}, is the amount of heat required to raise the temperature of a kilogram of water 1 degree at 20° C. (that is, from half a degree below 20° to half a degree above). The large calorie at any other temperature is here designated by C_t, and is the amount of heat required to raise the temperature of a kilogram of water 1 degree at the temperature t.

Since the specific heat of water is nearly a minimum at 20° C., our standard calorie, C_{20}, is nearly always less than C_t. This difference is ordinarily neglected. But in the alcohol-test experiments and in the experiment with a human subject to be described later, this difference is, relatively speaking, not inconsiderable. In Table VI. the specific heat of water is given for different temperatures between 0° and 31°. The figures are based upon the experiments of Rowland,[1] Bartoli and Stracciati,[1] Griffiths,[1] and Ludin.[2] The differences in the results obtained by these investigators at temperatures below 22° are very small, and, in view of the care with which their experiments were made, we do not believe that the estimates of this table can be far enough from the truth materially to diminish their value for the present purpose.

It will be seen that at 0° the calorie (C_0) is nearly 1 per cent. greater than C_{20}, and that C_{10} is about three parts in a thousand greater than C_{20}.

The results of all the combustions by the bomb calorimeter, as

[1] See p. 00 above.

[2] Inaug. Diss., Zurich, 1895; cited by Longuinine, Bestimmung der Verbrennungswärme, Berlin, 1897, p. 17. See discussion of results of experiments on the specific heat of water on pp. 12–20 of this valuable treatise.

TABLE VI.

Specific heat of water at different temperatures referred to that at 20° C. as unity.

Temp. °C.	Specific heat.	Temp. °C.	Specific heat.	Temp. °C.	Specific heat.	Temp. °C.	Specific heat.
0	1.0090	8	1.0039	16	1 0007	24	.9998
1	1.0083	9	1.0034	17	1.0004	25	.9998
2	1.0076	10	1.0029	18	1.0002	26	.9998
3	1.0069	11	1.0024	19	1.0001	27	.9999
4	1.0062	12	1.0020	20	1.0000	28	.9999
5	1.0056	13	1.0016	21	.9999	29	1.0000
6	1.0050	14	1.0013	22	.9998	30	1.0001
7	1.0044	15	1.0010	23	.9998	31	1.0002

well as the measurements by the respiration calorimeter, are to be expressed in terms of C_{20}. To do this it is necessary to know the mean specific heat of water for the range of temperature employed in any given experiment. For example, if water is warmed from 3° to 17° in passing through the absorbed pipes of the respiration calorimeter, the result will be in terms of $C_{(3\text{-}17)}$; that is, in terms of the mean calorie from 3° to 17°. This is 1.0032 times as great as the standard calorie C_{20}, whereas C_{10}, the calorie for the mean temperature, is only 1.0029 times as great as C_{20}. In other words, the mean specific heat from 3° to 17° is 1.0032 times the specific heat at 20°, whereas the specific heat at 10°, the mean temperature, is 1.0029 times that at 20°. The difference between the mean specific heat for this range of temperature and the specific heat of the mean temperature is therefore appreciable; this is of course because the variation of the specific heat is not linear. We have accordingly calculated the mean specific heat of the water in every case for the range of temperatures employed, and expressed the heat measured in terms of C_{20}. In the tables this range of temperature is given in the fourth column, and the heat in terms of C_{20} in the fifth column.

4. THE LATENT HEAT OF WATER VAPOR.

The value commonly used for the latent heat of vaporization of water at different temperatures is that given by Regnault[1] as the result of his classical investigations half a century ago. Reg-

[1] Mém. Acad. Roy. Sci. Inst. France, 21 (1847), pp. 635–728.

nault's formula, which expresses quite closely the results obtained from 63° to 195°, is $H = 606.5 + .305t$. H represents what Regnault called the "total heat" of steam or water vapor, at the temperature t; that is, it is the number of small calories required to raise a gram of water from 0° C. to any temperature t and then completely vaporize it at that temperature. For $t = 100°$, this gives $H = 637$. The "latent heat of vaporization" at any temperature, as distinguished from the "total heat" at the same temperature, is the number of units of heat required to evaporate the water after it has been brought to the temperature of evaporation. Hence at 100° C. the latent heat of vaporization is 537 calories. The formula for the latent heat at any temperature t is

$$L = H - t$$
$$\therefore L = 606.5 + .305\,t - t$$
$$\text{or } L = 606.5 - .695\,t.$$

Regnault's value for the latent heat of steam at 100° is abundantly confirmed by later researches.

At lower temperatures than 63°, however, the method employed was inapplicable because the change of state from water to steam at low pressure is irregular and explosive; hence a different method became necessary. The process was accordingly reversed, and water contained in a small reservoir inside a much smaller calorimeter than that previously used was evaporated at reduced pressure. The heat absorbed in the vaporization of the water (about 5 grams at each experiment) was then determined from the lowering of the temperature of the water which surrounded the vessel in which the evaporation took place. The results, as Regnault himself points out, were subject to comparatively large experimental errors, and were very discordant. The mean results over a range of temperature from 0° to 16° are, however, fairly well represented by the formula given above for higher temperatures.

Starkweather,[1] in a critical review of various determinations of the latent heat of water vapor, quotes the work of Dieterici,[2] Griffiths,[3]

[1] "Concerning Regnault's calorie and our knowledge of the specific volumes of steam." Amer. Jour. Sci., 7 (1899), p. 13.

[2] Ann. Phys. u. Chem. (Wiedemann), 38, 1889.

[3] Phil. Trans. Roy. Soc. (London), A, 1895.

and Svensson,[1] as the best that has been done at low pressure and temperatures. Dieterici and Svensson determined with an ice calorimeter the latent heat of water vapor at 0° C. Dieterici's work is very carefully done, and his result is $L = 598.9$ at 0° C., in terms of the quantity of heat required to raise 1 gram of water from 15° to 16° as the unit of heat. Svensson's result agrees very closely with this, being 599.9. Griffiths measured the latent heats of water vapor between 25° and 50° C., and succeeds in representing them by the formula $L = 596.73 - .601t$. Extrapolating to 0° and 100°, this agrees quite well with Dieterici at 0° and Regnault and others at 100°, being 596.73 at 0° and 536.6 at 100°. None of these experiments, however, have been carried out at the temperature commonly used in the respiration calorimeter—*i. e.*, 20° C. Hence we are obliged to deduce the latent heat of vaporization at 20° from formulas given by experiments at other temperatures.

Using Regnault's original formula, from which $L = 606.5 - .695\ t$ we get $L = 592.6$ at 20° C. Using Starkweather's modification of this formula, which agrees with Regnault's results at temperatures between 63° and 100° better than Regnault's, viz, $L = 598.9 - .558\ t - .00064t^2$, L becomes 587.5 at 20° C. Using Griffith's formula above as quoted $L = 584.5$ at 20° C.

Some preliminary experiments have been made with the respiration calorimeter, with a view of determining this quantity L, *i. e.*, the latent heat of vaporization of water at 20°, under the circumstances of the respiration experiment. The results so far have not, however, given a satisfactory value for this quantity. We have therefore taken 592 as a provisional value for the calculation of the experiments reported in this article, and it would appear that this is probably within 1 per cent. of the truth, although it is possibly more than one per cent. too great. Further investigations at 20° are necessary in order to fix the value more precisely.

5. Description of the Test Experiments.

Various substances have been used by experimenters with different forms of respiration apparatus for burning within the chamber

[1] Beiblätter, Ann. Phys. u. Chem. (Wiedemann), 20, p. 356.

in order to deliver known quantities of carbon dioxid and water for the purpose of testing the accuracy of the apparatus and methods.

The chief difficulty is to find substances which can be so burned as to insure complete oxidation of the carbon and hydrogen. The first attempts in this laboratory with the combustion of alcohol were unsatisfactory because the oxidation of the alcohol was incomplete, and a considerable amount of experimenting was necessary in order to learn the conditions under which complete combustion could be secured. As the result a small lamp, such as is ordinarily used with kerosene for illuminating purposes, has been employed. By proper arrangement of wick and chimney the alcohol is burned so completely that no traces of volatilized alcohol, acetone, aldehyde, carbon monoxid, or other substances capable of yielding carbon dioxid upon heating with oxygen could be found among the products of combustion.

A considerable number of alcohol test experiments were made. The individual experiments included from one to seven periods or "runs" of approximately six hours each.

The lamps used in these tests contain approximately 380 grams of alcohol. By adjusting the wick the rate of burning was so regulated as to give off carbon dioxid and water in the desired amounts.

In Table VII. are summarized the principal results of all the alcohol check tests which were made up to the ninth, except the preliminary tests, in which the methods of manipulation were being worked out, and two tests the completion of which was prevented by accident. These individual experiments continued from 5 to 78 hours each, the total time being 317 hours. The rate of burning of the alcohol ranged from 10 to 27 grams per hour, and the strength of the alcohol from 90.26 to 90.77 per cent. Most of the tests were made in alternation with experiments with men, the object being to test the accuracy of the apparatus before and after each of the latter experiments.

It will be noticed that in No. 4 the determination of carbon dioxid was unsatisfactory, though we were unable to decide whether the error was due to imperfect sampling or other cause. The result is not included in the average of Table VII. The measurement of the

TABLE VII.

Summary of nine alcohol test experiments

No.	Date.	Duration.		Alcohol burned.	Carbon dioxid.			Water.			Heat.		
					Required.	Found.	Ratio of amount found to amount required.	Required.	Found.	Ratio of amount found to amount required.	Required.	Found.	Ratio of amount found to amount required.
	1897.	*h.*	*m.*	*Grams.*	*Grams.*	*Grams.*	*Per cent.*	*Grams.*	*Grams.*	*Per cent.*	*Calories.*	*Calories.*	*Per cent.*
1	April 27-29	52	31	955.4	1,657.2	1,657.6	100.0	1,106.1	1,109.7	100.3	6,129.0	6,077.1	99.15
2	May 10-11	29	56	798.8	1,385.6	1,384.4	99.9	924.8	925.0	100.0	5,124.2	5,167.1	100.84
3	May 26-27	33	50	505.4	876.7	887.8	101.3	585.1	627.9	[107.3]	3,242.1	3,217.7	99.25
4	October 27-28	34	33	797.7	1,384.8	1,335.7	[96.5]	925.7	1,007.9	[108.8]	5,120.5	5,141.5	100.41
5	November 2-3	35	09	788.2	1,365.1	1,376.7	100.8	912.3	920.8	100.9	5,048.4	5,050.0	100.03
6	December 2	11	39	245.3	423.1	417.6	98.6	283.7	287.5	101.3	1,564.8	1,556.8	99.48
	1898.												
7	January 6	5	50	112.2	193.5	193.5	100.0	129.8	131.3	101.2	715.7	731.1	102.15
8	January 24-27	77	57	1,607.8	2,784.4	2,769.7	99.5	1,860.8	1,881.6	101.1	10,294.7	10,268.5	99.74
9	May 9	35	55	699.7	1,206.9	1,198.9	99.4	809.3	807.9	99.8	4,463.4	4,466.0	100.05
	Total [1]	317	20		9,892.5	9,886.2	99.9	6,026.8	6,063.8	100.6	41,702.8	41,675.8	99.93

[1] Omitting the carbon dioxid and water in test No. 4 and the water in test No. 3.

heat was, however, very close to the theoretical value. Almost immediately after the close of this test another was made, in which all the results were very closely in accord with the theoretical values. If the determinations of carbon dioxid in test No. 4 are omitted, the maximum variation of the amounts determined from the theoretical amounts given off by the combustion of the alcohol was 1.4 per cent. and the average variation only 0.1 per cent., or 1 part in 1,000 from the theoretical.

In tests Nos. 3 and 4 the determinations of water were unsatisfactory. We are inclined to attribute the errors to variations in the amounts of water condensed upon the absorbers. These results, like that for carbon dioxid in No. 4, are omitted from the averages. Omitting the determinations of water in tests Nos. 3 and 4, the maximum variation, as actually determined from the theoretical amount, was 1.2 per cent. and the average variation only 0.6 per cent. In test No. 7 the proportion of heat measured was larger than usual. It will be observed, however, that this test continued only through one period of six hours. Some time is required to get the apparatus in temperature equilibrium, and the heat measurements of the first experimental period are frequently incorrect on this account. The omission of this experiment would not materially affect the total averages. Omitting this experiment, the maximum variation of the heat actually measured from the theoretical amount was 0.8 per cent. and the average variation only 0.1 per cent.

6. SUMMARY OF TEST EXPERIMENTS.

The accuracy of the methods for the determination of carbon dioxid, water and heat was tested by heat generated in the chamber by passing an electric current through a resistance coil and by burning ethyl alcohol within the chamber. In the electrical tests the measurements of heat generated and found were practically identical, the differences between the theoretical and actual results averaging about 0.1 per cent—that is, about 1 part in 1,000. In the alcohol tests the average amounts found by actual experiment were: For carbon, 99.9 per cent.; hydrogen, 100.6 per cent.; and heat, 99.9 per cent., respectively, of the theoretical amounts.

The determinations of carbon dioxid and water made by burning

large quantities of alcohol in the respiration chamber agree reasonably well with each other and with the theoretical amounts. The variations, indeed, are not greater than are found in ordinary laboratory experience when alcohol is burned in the combustion furnace by the usual methods of organic analysis.

The agreement thus shown to exist between the results given by the respiration calorimeter and the bomb calorimeter for the heat of combustion of alcohol, is also very satisfactory when the great difference in the circumstances of the experiments is taken into consideration. In the bomb calorimeter a fraction of a gram of alcohol is absorbed in a small block of cellulose or inclosed in a gelatin capsule and placed in a steel cylinder of perhaps half a liter capacity, which is filled with oxygen at a pressure of 20 atmospheres. An electric current passes through a fine iron wire, melts the latter, and ignites the alcohol, which is completely oxidized in an instant. The increase of temperature of the bomb and the water in which it is immersed gives the quantity of heat evolved, and from that the heat of combustion of a gram of alcohol is computed. In the respiration calorimeter, on the other hand, the alcohol contained in a small lamp burns quietly for many hours in a chamber ten thousand times as large as the bomb. Oxygen is supplied by a continuous current of air pumped through the apparatus, and a considerable portion of the heat of combustion is carried away in the latent heat of the water vapor produced. The sum of the heat measured by the calorimeter and the latent heat of the water vapor collected gives the total heat produced by the burning alcohol. That the average of a series of nine experiments should vary less than 0.1 per cent. from the average of the determinations with the bomb calorimeter seems a gratifying result. Taken in connection with the electrical tests and the determinations of carbon dioxid and water already given, the results show that the respiration calorimeter is an instrument of precision and abundantly capable of doing the work for which it was designed.

IV.

EXPERIMENTS WITH MAN.

The apparatus previously described was designed exclusively for experiments with human subjects. There were several reasons for beginning with men rather than with domestic animals. The study of human nutrition is very important. In the earlier development of the work, when many difficulties were to be overcome, it was very desirable to have inside the apparatus an intelligent person who could make and record important observations during the experiment, rather than an animal whose movements, even, could not be controlled. Indeed, the most advantageous way to develop methods and apparatus for experiments with animals is through such preliminary experience with men. The results of the experience thus far gained are now being utilized both here and elsewhere in planning apparatus and methods to be used not only with small animals, as rabbits, sheep and dogs, but also with larger animals, as horses, oxen and cows.

Plan of Experiments.

The diet chosen for any given experiment is followed for eight days, of which the last four constitute the period of the experiment proper. In the first or preliminary period, extending through four days, the analyses of feces and urine are made, the data thus sufficing for a digestion and nitrogen metabolism experiment. On the evening of the fourth day the subject enters the respiration chamber, though the actual respiration calorimeter experiment does not begin until 7 o'clock on the morning of the fifth day. The night sojourn in the apparatus suffices to get the temperature of the apparatus and its content of carbonic acid and water into equilibrium, so that accurate measurements may begin with the morning of the fifth day and continue until 7 o'clock on the morning of the ninth day, thus making the duration of this experiment exactly four days. As in the test experiments already reported the determinations of carbon dioxid, water vapor and heat are made in six-hour periods, so that the complete data for an experiment show the total amounts of these compounds given off from the body during the periods ending

at 1 p. m., 7 p. m., 1 a. m. and 7 a. m. of each day of the experiment. The urine is also collected and the nitrogen determined for corresponding periods.

The accompanying schedule shows the daily routine followed in the experiment here reported, which was carried out in January, 1898.

TABLE VIII.

Daily program of the experiment.

7.00 a. m.	Rise.	1.15 p. m.	Dinner.
	Pass urine.	4.00 p. m.	Drink 200 grams water.
	Weigh self, stripped.	6.30 p. m.	Supper.
	Weigh absorbers.	7.00 p. m.	Pass urine.
	Collect drip.		Collect drip.
7.45 a. m.	Breakfast.		Weigh absorbers.
10.30 a. m.	Drink 200 grams water.	10.00 p. m.	Drink 200 grams water.
1.00 p. m.	Pass urine.		Weigh self, stripped.
	Collect drip.		Retire.
	Weigh absorbers.	1.00 a. m.	Pass urine.

In this experiment the subject was as quiet as practicable. In the four days of the preliminary period he moved about but little and engaged in no considerable amount of either muscular or mental labor. During the four days passed in the chamber he was likewise quiet. The only muscular work done was that involved in dressing, putting up and taking down the folding chair, table, and bed, weighing himself and the absorbers, taking his meals, and caring for the excreta. He passed a large part of the time in reading and sleeping.

The heats of combustion of the food, feces, and urine were determined in the usual manner by means of the bomb calorimeter.

The determinations of carbon and hydrogen were made in the usual manner. The partially dried samples of food, feces and urine were burned with cupric oxid with the aid of a current of oxygen, and the water absorbed by sulphuric acid, the carbon dioxid by potassium hydroxid.

The percentage composition and heats of combustion of the food used in the experiment here described, and of the feces for the same experiment is shown in Table IX.

TABLE IX.

Composition of food materials, etc.

Food materials, etc.	Nitrogen.	Carbon.	Hydrogen.	Water.	Protein (N × 6.25).	Fat.	Carbohyd'tes	Ash.	Heats of combustion per gram.
	P. ct.	*P. ct.*	*P. ct.*	*P. ct.*	*P. ct.*	*P. ct.*	*P. ct.*	*P. ct.*	*Cals.*
Beef, cooked.	4.10	16.35	2.25	67.3	25.6	5.4		1.56	1.928
Butter.	.19	62.68	10.27	10.2	1.2	84.8		3.80	7.761
Milk, skimmed.	.52	4.04	.57	90.7	3.3	.1	5.2	.78	.393
Bread.	1.34	24.53	3.54	44.7	8.4	.2	44.3	2.44	2.400
Breakf. food, wheat.	1.58	41.32	5.78	7.5	9.9	1.6	77.7	3.26	4.071
Ginger snaps.	.96	44.45	6.48	5.2	6.0	9.5	75.6	3.70	4.358
Breakf. food, maize.	1.78	44.34	6.45	5.6	11.1	8.7	71.1	3.46	4.444
Sugar.		42.10	6.48				100.0		3.960
Feces.	1.19	12.60	1.74	72.9	7.4	3.9	11.4	4.40	1.343

EXPLANATION OF THE TABLES.

The results obtained in this experiment are shown in the following tables. Only such of the data are given here as it is believed will be sufficient for a clear understanding of the experiments.

The daily menu was as follows:

TABLE X.

Daily menu.

Breakfast.	Grams.	Dinner.	Grams.	Supper.	Grams.
Cooked beef.	100	Cooked beef.	150	Butter.	15
Butter.	15	Butter.	20	Skimmed milk.	390
Skimmed milk.	160	Skimmed milk.	210	Bread.	25
Bread.	25	Bread.	50	Breakfast food,	
Breakfast food,		Breakfast food,		wheat.	75
maize.	50	wheat.	50	Ginger snaps.	60
Sugar.	25	Sugar.	25	Sugar.	30
Coffee, about.	300	Coffee, about.	300	Coffee, about.	300

Table XI. shows the amount of carbon dioxid eliminated. The table shows the number of liters of air in the ventilating air current and the milligrams per liter of carbon dioxid in the incoming and in the outgoing air during each six-hour period of the experiment. From these data the total excess of carbon dioxid in the outgoing over that from the incoming air is calculated. The analysis of the

residual air in the apparatus at the close of each period gives data for the calculation of the increase or decrease of the carbon dioxid in the chamber at the end as compared with the beginning of each period. Applying this correction the actual elimination of carbon dioxid by the subject is obtained for each six-hour period. From this is calculated the total weight of carbon exhaled.

TABLE XI.

Record of carbon dioxid.

Day.	Period.	Ventilation: number of liters of air.	Carbon dioxid per liter.			Total excess in outgoing air.	Correction for carbon dioxid in apparatus.	Corrected weight carbon dioxid exhaled by subject.	Total weight carbon exhaled in carbon dioxid.
			In incoming air.	In outgoing air.	Excess in outgoing air.				
		Liters.	*Mg.*	*Mgs.*	*Mgs.*	*Grams.*	*Grams.*	*Grams.*	*Grams.*
1	7 a.m. to 1 p.m.	25,712	0.580	9.556	8.976	230.8	+ 3.1	233.9	63.8
	1 p.m. to 7 p.m.	25,987	.563	9.686	9.123	237.1	+14.9	252.0	68.7
	7 p.m. to 1 a.m.	26,785	.622	8.997	8.375	224.3	−14.7	209.6	57.2
	1 a.m. to 7 a m.	26,065	.593	5.891	5.298	138.1	− 2.5	135.6	37.0
	Total.....	104,549	.....	....	....	830.3	+ .8	831.1	226.7
2	7 a.m. to 1 p.m.	27,057	.571	8.750	8.179	221.3	+11.3	232.6	63.4
	1 p.m. to 7 p.m.	25,878	.560	9.109	8.549	221.3	+ 5.3	226.6	61.8
	7 p.m. to 1 a.m.	26,652	.612	9.556	8.944	238.4	−12.6	225.8	61.6
	1 a.m. to 7 a.m.	26,011	.629	5.765	5.136	133.6	− 5.2	128.4	35.0
	Total.....	105,598	.....	.....	....	814.6	− 1.2	813.4	221.8
3	7 a.m. to 1 p.m.	26,342	.624	9.139	8.515	224.3	+19.2	243.5	66.4
	1 p.m. to 7 p.m.	26,492	.721	9.349	8.628	228.6	+ .3	228.9	62.4
	7 p.m. to 1 a.m.	26,147	.722	9.326	8.604	225.0	−10.6	214.4	58.5
	1 a.m. to 7 a.m.	25,163	.727	5.917	5.190	130.6	− 8.1	122.5	33.4
	Total.....	104,144	....	.....	.....	808.5	+ .8	809.3	220.7
4	7 a.m. to 1 p.m.	26,427	.710	8.973	8.263	218.4	+21.2	239.6	65.3
	1 p.m. to 7 p.m.	25,731	.735	9.525	8.790	226.2	+ .3	226.5	61.8
	7 p.m. to 1 a.m.	26,046	.636	10.021	9.385	244.5	−14.0	230.5	62.9
	1 a.m. to 7 a m.	26,338	.612	5.870	5.258	138.5	− 9.1	129.4	35.3
	Total.....	104,542	.....	.....	.....	827.6	− 1.6	826.0	225.3
	Total, 4 days...	418,833	.....	.....	....		.....	3,279.8	894.5
	Av. per day....	104,708	....	.....	.....	.....	.....	819.9	223.6

Table XII. shows the amount of water exhaled. It indicates the number of liters of air in the ventilating air current and the milligrams per liter of water vapor in the incoming air and in the

outgoing air after passing the freezers, which latter condense the major portion of the water vapor in the outgoing air current. From these data, together with the amount of water condensed in the freezers, the amount condensed in the chamber as "drip," and the determinations of residual water vapor in the chamber, is computed the total water exhaled by the subject.

TABLE XII.

Record of water.

Day.	Period.	Ventilation; number of liters of air.	Water per liter.			Total excess in outgoing air.	Condensed in freezers.	Condensed in chamber.	Correction for water vapor in chamber.	Total water exhaled.
			In incoming air.	In outgoing air.	Excess in outgoing air.					
		Liters.	*Mg.*	*Mgs.*	*Mg.*	*Grams.*	*Grams.*	*Grams.*	*Grams.*	*Grams.*
1	7 a.m. to 1 p.m.	25,712	0.895	1.301	0.406	10.4	207.5	14.0	−13.5	218.4
	1 p.m. to 7 p.m.	25,987	.816	1.258	.442	11.5	230.2	22.0	+15.3	279.0
	7 p.m. to 1 a.m.	26,785	.792	1.184	.392	10.5	238.1	−18.0	+ 3.1	233.7
	1 a.m. to 7 a.m.	26,065	.731	1.143	.412	10.7	223.0	.....	− 6.9	226.8
	Total.....	104,549	.....	.....	.....	43.1	898.8	18.0	− 2.0	957.9
2	7 a.m. to 1 p.m.	27,057	.737	1.187	.450	12.2	212.0	3.0	+ .5	227.7
	1 p.m. to 7 p.m.	25,878	.821	1.232	.411	10.6	213.7	− 1.0	+ 2.2	225.5
	7 p.m. to 1 a.m.	26,652	.749	1.221	.472	12.6	230.7	−21.0	+ 4.6	226.9
	1 a.m. to 7 a.m.	26,011	.723	1.219	.496	12.9	208.4	.....	− 9.7	211.6
	Total	105,598	....	.....	.....	48.3	864.8	−19.0	− 2.4	891.7
3	7 a.m. to 1 p.m.	26,342	.764	1.320	.556	14.6	202.1	13.1	+ 3.8	233.6
	1 p.m. to 7 p.m.	26,492	.772	1.206	.434	11.5	208.3	4.0	+ .5	224.3
	7 p.m. to 1 a.m.	26,147	.740	1.088	.348	9.1	230.6	− 8.0	+10.5	242.2
	1 a.m. to 7 a.m.	25,163	.799	1.271	.472	11.9	209.0	.....	−15.1	205.8
	Total.....	104,144	....	.....	.....	47.1	850.0	9.1	− .3	905.9
4	7 a.m. to 1 p.m.	26,427	.793	1.293	.500	13.2	209.2	41.8	+ 3.9	268.1
	1 p.m. to 7 p.m.	25,731	.726	1.273	.547	14.1	206.2	37.8	+ 2.3	260.4
	7 p.m. to 1 a.m.	26,046	.776	1.128	.352	9.2	235.5	− 6.7	+ 5.4	243.4
	1 a.m. to 7 a.m.	26,338	.715	1.109	.394	10.4	234.6	4.2	−11.6	237.6
	Total	104,542	.....	.....	.....	46.9	885.5	77.1	.....	1,009.5
	Total, 4 days..	418,833	.....	.....	.....	185.4	3,499.1	85.2	− 4.7	3,765.0
	Av. per day...	104,708	...	....	.....	.....	.....	....	....	941.3

The calorimetric results obtained in this experiment are shown in Table XIII. The details of the table and moethds of computation are explained in Bulletin 63 of the Office of Experimental Stations, U. S. Dept. of Agriculture.

TABLE XIII.

Summary of calorimetric measurements

Day.	Period.	Heat measured.	Change of temperature of calorimeter.	Capacity correction.	Correction due to temperature of food and dishes.	Corrected heat.	Water vaporized.	Equivalent heat of water vaporized.	Total heat.
		Cals.	°*C.*	*Cals.*	*Cals.*	*Cals.*	*Grams.*	*Cals.*	*Cals.*
1	7 a.m. to 1 p.m.	547.9	+0.12	+ 7.2	− 22.2	532.9	217.9	129.0	661.9
	1 p.m. to 7 p.m.	556.6	− .12	− 7.2	− 37.5	511.9	241.7	143.1	655.0
	7 p.m. to 1 a.m.	472.7	− .05	− 3.0	0	469.7	248.6	147.1	616.8
	1 a.m. to 7 a.m.	277.3	− .03	− 1.8	0	275.5	233.7	138.4	413.9
	Total	1,854.5	.....	− 4.8	− 59.7	1,790.0	.....	557.6	2,347.6
2	7 a.m. to 1 p.m.	490.7	+ .04	+ 2.4	− 22.9	470.2	224.2	132.7	602.9
	1 p.m. to 7 p.m.	489.2	+ .08	+ 4.8	− 28.3	465.7	224.5	132.9	598.6
	7 p.m. to 1 a.m.	497.4	− .15	− 9.0	0	488.4	243.3	144.1	632.5
	1 a.m. to 7 a.m.	289.2	+ .15	+ 9.0	0	298.2	221.3	131.0	429.2
	Total.....	1,766.5	.....	+ 7.2	− 51.2	1,722.5	.....	540.7	2,263.2
3	7 a.m. to 1 p.m.	524.6	+ .08	+ 4.8	− 21.2	508.2	216.7	128.3	636.5
	1 p.m. to 7 p.m.	512.1	+ .08	+ 4.8	− 31.8	485.1	219.8	130.1	615.2
	7 p.m. to 1 a.m.	486.6	− .18	−10.8	0	475.8	239.7	141.9	617.7
	1 a.m. to 7 a.m.	296.1	+ .10	+ 6.0	0	302.1	220.9	130.8	432.9
	Total.....	1,819.4	.....	+ 4.8	− 53.0	1,771.2	.....	531.1	2,302.3
4	7 a.m. to 1 p.m.	503.3	− .05	− 3.0	− 21.3	479.0	222.4	131.7	610.7
	1 p.m. to 7 p.m.	536.4	+ .05	+ 3.0	− 29.2	510.2	220.3	130.4	640.6
	7 p.m. to 1 a.m.	497.5	− .10	− 6.0	0	491.5	244.7	144.9	636.4
	1 a.m. to 7 a.m.	290.7	+ .04	+ 2.4	0	293.1	245.0	145.0	438.1
	Total.....	1,827.9	.....	− 3.6	− 50.5	1,773.8	.....	552.0	2,325.8
	Total, four days	7,268.3	.....	+ 3.6	−214.4	7,057.5	.....	2,181.4	9,238.9
	Av. per day	1,817.1	.....	+ .9	− 53.6	1,764.4	.....	545.3	2,309.7

As has been previously mentioned, the urine was collected in six-hour periods beginning at 7 a. m. (the hour of beginning and ending the experiment). The amount of nitrogen in the urine from 7 a. m. of one day to 7 a. m. next day is taken as a measure of the protein metabolized during this period. Of course this makes no allowance for the nitrogen lag, *i. e.*, the period between the metabolism of the nitrogen and its excretion, but it was considered that the error thus introduced might be ignored, since the subject had been living on the same diet and had had the same exercise from day to day for the experimental period of four days and the previous four days

of the preliminary experiment also. The amount of nitrogen in the urine for 12 to 24 hours after the close of the experiment was also determined in six-hour periods, although these values are not recorded here. In Table XIV. is summarized the amount of urine for each day of the experiment, together with the percentage and amounts of nitrogen and carbon, and the heat of combustion per gram and the total heat of combustion of the urine.

TABLE XIV.

Amount and composition of urine.

	Amount.	Nitrogen.		Carbon.		Heat of combustion.	
						Per gm.	Total.
	Grams.	*Per cent.*	*Grams.*	*Per cent.*	*Grams.*	*Calories.*	*Calories.*
First day.	1,855.3	1.01	18.75	0.69	12.80	0.082	152
Second day.	1,977.6	.95	18.75	.65	12.79	.081	160
Third day.	1,510.6	1.21	18.28	.83	12.50	.095	143
Fourth day.	1,358.9	1.32	17.89	.90	12.19	.102	139
Total, four days.	6,702.4	.....	73.68	.....	50.28	.....	594
Average per day.	1,675.6	.....	18.42	.....	12.57	.....	149

The data previously given serve for the calculation of Tables XV. and XVI., which show the gain or loss of nitrogen, carbon, protein and fat and the comparison of the estimated heat of the material oxidized in the body with the heat actually measured in this experiment. The calculations are partially explained by the letters and algebraic formulas at the tops of the columns. Thus column *d* indicates the gain or loss of nitrogen, and is computed by adding the amounts in the feces and urine, as shown in columns *b* and *c*, and subtracting this sum from the amount in the food as shown in column *a*. Protein contains, approximately, 16 per cent. of nitrogen, and hence by multiplying the nitrogen loss by 6.25 we get the quantity of protein of the body which must have been oxidized; this is given in column *e*.

The numbers in columns *f*, *g* and *h* are the results of direct chemical analysis of samples of food, feces and urine, the details of which are not here reported. The numbers in column *i* are taken from Table XI., and column *j* gives the net gain of carbon during each day of the experiment.

As protein contains, approximately, 53 per cent. of carbon, the amount of carbon yielded by the daily consumption of body protein is computed by multiplying the numbers of column *e* by .53; these results are given in column *k*. But there was a net gain to the body of carbon, which is explained by the inference that there was more carbon stored in fat accumulated than was lost in protein

TABLE XV.

Income and outgo of nitrogen and carbon, with estimated gain or loss of protein and fat.

	Nitrogen.					Carbon.								
	a. In food.	*b.* In feces.	*c.* In urine.	*d.* Net loss $= a - (b + c)$.	*e.* Protein loss $= d \times 6.25$.	*f.* In food.	*g.* In feces.	*h.* In urine.	*i.* In respiratory products.	*j.* Net gain $= f - (g + h + i)$.	*k.* Carbon in protein lost $= e \times 0.53$	*l.* Carbon in fat gained $= f - k$.	*m.* Fat gain $= l \div 0.765$.
	Gms.	*Gms.*	*Gms.*	*Gms.*	*Gms.*	*Gms.*	*Gms.*	*Gms.*	*Gms.*	*Gms.*	*Gms.*	*Gms.*	*Gms.*
First day.	19.1	1.2	18.7	−0.8	− 5.0	261.5	13.3	12.8	226.7	+ 8.7	−2.7	+11.4	+14.9
Second day.	19.1	1.3	18.8	−1.0	− 6.3	261.6	13.4	12.8	221.8	+13.6	−3.3	+16.9	+22.1
Third day.	19.1	1.2	18.3	− .4	− 2.5	261.5	13.3	12.5	220.7	+15.0	−1.3	+16.3	+21.3
Fourth day.	19.1	1.3	17.9	− .1	− 0.6	261.6	13.4	12.2	225.3	+10.7	− .3	+11.0	+14.4
Total.	76.4	5.0	73.7	−2.3	−14.4	1,046.2	53.4	50.3	894.5	+48.0	−7.6	+55.6	+72.7
Average per day.	19.1	1.2	18.4	− .6	− 3.6	261.5	13.3	12.6	223.6	+12.0	−1.9	+13.9	18.2

oxidized. We get the quantity of carbon stored in body fat by adding the numbers of columns *j* and *k*, and the sums are shown in column *l*. Finally, since fat contains about 76.5 % of carbon, we obtain the quantities of fat daily stored in the body by dividing the numbers of column *l* by .765, and these quantities are shown in column *m*.

In Table XVI. the values given in columns *n*, *o* and *p* are actual heats of combustion as determined by the bomb calorimeter. The values in columns *q* and *r* are obtained by the use of the factors 5.5 and 9.4 representing the average heat of combustion of 1 gram of protein and fat, respectively. The values in column *t* are obtained from Table XIII.

TABLE XVI.

Income and outgo of energy.

	Energy (heats of combustion).							Ratio of heat measured to that of material actually oxidized. $t \div s$.
	Actually determined.			Estimated.				
	n. Of food.	*o*. Of feces.	*p*. Of urine	*q*. Of protein lost $=e \times 5.5$.	*r*. Of fat gained $=m \times 9.4$	*s*. Of material actually oxidized in body $=n-(o+p+q+r)$.	*t*. Heat actually measured.	
	Calories.	*Calories.*	*Calories.*	*Calories.*	*Calories.*	*Calories.*	*Calories.*	*Per Cent.*
First day.	2,717	142	152	−27	+140	2,310	2,348	
Second day.	2,717	142	160	−35	+208	2,242	2,263	
Third day.	2,717	142	143	−14	+200	2,246	2,302	
Fourth day.	2,717	142	139	− 3	+135	2,304	2,326	
Total.	10,868	568	594	−79	+683	9,102	9,239	.....
Av. per day.	2,717	142	149	−20	+171	2,275	2,310	101.5

RESULTS OF THE EXPERIMENT WITH A HUMAN SUBJECT.

Table XV. shows that the subject under experiment eliminated daily on an average 19.7g. of nitrogen while he received only 19.1 g. of nitrogen in his food. This daily loss of nitrogen is taken to indicate a daily metabolism of protein, the heat of combustion of which is given in column *q* of table XVI. The energy of the material actually oxidized in the body, as given in column *s*, is obtained as follows : Taking the first day as an example, we subtract the heat equivalent of feces

and urine, 294 calories, from the total heat value of the food, 2,717. This leaves 2,423 calories. But 140 was stored in the body as fat while 27 was realized by combustion of body protein. Hence 113 calories was stored over and above that derived from body protein, and subtracting 113 from 2,423 we have 2,310 calories as the net amount of energy that should have appeared as heat when all mechanical work of the body, internal and external, has been converted into heat. The average for the four days was 2,275, while the average quantity of heat measured by the calorimeter, including the latent heat of water vapor carried away in the air current, as shown in Table XIII., was 2,310 calories. This is a difference of 35 calories, or 1.5 %.

In later experiments, about to be reported, the differences between income and outgo of energy average somewhat nearer to the theoretical; in other words, in these more accurate experiments not far from 99 per cent. of the potential energy of the material metabolized and oxidized in the body is accounted for in the kinetic energy given off in the forms of heat and external muscular work.

In how far this fairly close agreement is due to a counterbalancing of errors it is impossible to say. But in view of the physiological difficulties in the way of absolutely accurate results, and the evident possibility of minor errors in the purely chemical and physical determinations, and likewise in the factors used for computation, this agreement seems to us very satisfactory. We believe that it marks a stage in the development of the apparatus and methods sufficient to warrant extended series of experiments upon various questions connected with the laws of nutrition, and such experiments have been begun. Efforts are being made at the same time to eliminate part at least of the experimental errors. At present these efforts are chiefly in the direction of improvement of methods of sampling and analyzing the food materials and excretory products, and finding of minor sources of error in the determination of carbon, hydrogen and heat given off in the respiration chamber, and the direct determination of the oxygen of income and outgo. Minor alterations are also being made in the apparatus and the methods of its manipulation by which it is hoped that somewhat greater accuracy may be secured.

MECHANICAL EFFICIENCY OF A MAN.

In May, 1897, an experiment was carried out in which the subject under investigation performed a considerable amount of muscular work. This was converted into heat by means of a small dynamo belted to a stationary bicycle. The current generated in the dynamo passed through an incandescent lamp, and all the heat produced, either by friction or by the electric current, was measured by the calorimeter along with that given off by the body directly. This experiment was less accurate than the one reported above for two principal reasons: first, because the methods of sampling the food were then less accurate than in the later experiments, and second, because the large amount of heat evolved during working hours, and the sudden changes of rate from work to rest, made it impossible to measure the heat as accurately as in the "rest" experiment. The average number of calories measured daily was 3,726, the number which corresponds to the total material oxidized was 3,830; thus the difference was 2.7 per cent. of the latter amount. The amount of work done by the subject on the bicycle-dynamo was separately determined. This was done by adding to the energy of the current, as measured by a voltmeter and an ammeter, the energy required to overcome the friction of the bicycle-dynamo. The latter was determined by driving the apparatus by the dynamo used as a motor, and measuring the energy required.

The average work done was the equivalent of 256 large calories per day. This is about 40 watts for nearly eight hours, or 109,000 kilogrammeters, or 788,000 foot-pounds, or 394 foot-tons. The total energy actually yielded by the body, including that derived from the food and the body fat oxidized was 3,726 large calories daily. Dividing the work done by the total energy yielded by the body we have .07 or 7 per cent. as the mechanical efficiency.

But inasmuch as the energy received by the body is largely expended in the internal physiological work and in maintaining the temperature of the body, we should charge against the work done only the excess of energy absorbed in the work experiment over that required in a rest experiment. The average amount of energy used in several rest experiments with the same man has been about 2,500 calories. This subtracted from 3,726 leaves 1,226

as the excess of the work experiment. $256 \div 1{,}226 = .21$ Hence we have 21 per cent. as the efficiency of conversion of potential energy of food into mechanical energy.

The smaller value of the efficiency, namely 7 per cent. is greater than that of small steam engines, while the larger value, 21 per cent. equals or exceeds that of the best compound condensing engines with the highest efficiency boilers. A large part of the excess of heat in the work experiment is carried away in the latent heat of water vapor of perspiration. This may be likened to the latent heat of the expanded steam escaping from the cylinder of a steam engine. It is probable that other experiments may give a still larger value for the efficiency of man as a machine for converting the potential energy of food into mechanical energy. For man in this experiment was unused to the bicycle, and the work done by him was less than his maximum capacity, and was certainly very much less than the maximum capacity of an experienced bicyclist. As to the obscure but very interesting question of how this conversion is effected, whether it is a conversion of chemical potential energy directly into mechanical energy, or whether the material of food or muscle upon oxidation first yields heat which is subsequently converted into work by a process equivalent to that of a heat engine, or whether it is some other process, these experiments give no information.

General Summary.

The attempt has been made in the preceding pages to describe (1) a new form of respiration calorimeter and the methods of its use, and (2) several experiments in which the apparatus and methods have been employed.

The experiments here described had a two-fold purpose: To test the accuracy of the apparatus and methods, and to determine the balance of income and outgo of matter and energy in the body. They are preliminary to more extended research upon some of the fundamental problems of nutrition.

The name here used for the apparatus, "respiration calorimeter," is suggested by the fact that it is essentially a respiration apparatus with appliances for calorimetric measurements. As a respiration apparatus it is similar in principle to that of Pettenkofer.

The accuracy of the apparatus and of the methods for the determinations of carbon dioxid, water and heat was tested by heat generated in the chamber by passing an electric current through a resistance coil and also by burning ethyl alcohol within the chamber. In the electrical tests the measurements of heat generated and found were practically identical. In the alcohol tests the average amounts found by actual experiment were: For carbon, 99.9 per cent.; hydrogen, 100.6 per cent.; and heat, 99.9 per cent. of the theoretical amounts. It thus appears that this apparatus when used for the analysis of alcohol and the determination of its heat of combustion gives results nearly, if not quite, as accurate as are obtained by the ordinary laboratory methods which can be used only with small amounts.

The measurements of heat given off from the body of a man inside the chamber are so delicate that very slight bodily movements, such as rising from a chair or turning over in bed, are immediately noticed by the observer, who is constantly watching the galvanometer and thermometers.

The experiments with men, one of the earlier of which is here reported, were undertaken for the study of several problems. The question especially considered in this article is this: Is the energy given off from the body in the form of heat, or of heat and external muscular work, equal to the potential energy or heat of combustion of the material actually burned in the body? In other words, when the compounds of the food and the body—proteids, fats, and carbohydrates—are burned is their potential energy transformed into the equivalent kinetic energy and into forms which can be measured by the means here used? Or, to state the question more broadly, does the law of the conservation of energy obtain in the living organism?

In the experiment reported above the outgo apparently exceeds the income by about 35 calories per day. This quantity is really quite small. It would correspond to the potential energy of about 4 grams of body fat, or nearly the same weight of butter, or 9 grams of sugar, or 14 grams of bread.

The sources of error and uncertainty in these experiments can, it seems to us, be divided into two classes: those incident to the ex-

perimental work as such, and those due to uncertainty as to the physical, chemical and physiological constants used as factors in the computations.

So far as the sources of error are those of chemical and physical analyses and manipulation, the attempt is being made to reduce them to a minimum. In so far as they are of physiological origin and practically beyond control the attempt is made to eliminate them so far as practicable by long experimental periods and by duplication of the experiments.

Meanwhile it is safe to say that in view of the physiological uncertainties and sources of error and the probable incorrectness of some of the physical and chemical constants employed, the differences between the estimated income and measured outgo of energy in the experiments with men are not at all surprising.

In view of these defects and sources of error in methods and apparatus we would perhaps be unwarranted in assuming that the experiments thus far made completely demonstrate the application of the law of the conservation of energy in the human organism. They do, however, seem to us to be reasonably near to such demonstration.

It is certainly safe to assume that the principle followed in the experiments is correct, and that the apparatus and methods are accurate to the degree required for the experimental study of a large variety of the fundamental problems of biological chemistry and physics. Among these are the metabolism of energy and the production of heat by the body in the performance of its ordinary functions, as circulation, respiration and digestion; the relations of muscular and mental work to the metabolism of matter and energy; the demands of the body for nutriment under different conditions of work and rest, and under different conditions of health and disease; the duties performed by the different nutrients of food in supplying the needs of the body; and finally, the nutritive values of food materials and the amount and proportions best adapted to the needs of people of different classes, with different occupations, and in different conditions of life.

WESLEYAN UNIVERSITY, MIDDLETOWN, CONN., July, 1899.

NEW BOOKS.

Radiation, an Elementary Treatise on Electro-magnetic Radiation and on Röntgen and Cathode Rays. By H. H. FRANCIS HYNDMAN, B.Sc. Pp. 370. New York, The Macmillan Co.

The more or less arbitrary division of the subject of physics into different branches has usually resulted in leaving in the minds of elementary students a rather indefinite idea of the borderlands of these branches. In this book the reader will find much help in a subject which extends into the subjects of sound, heat, light and electricity. As Professor S. P. Thompson, in the preface to this book, says, "It is just these border regions which, neglected in the text-books, are the richest fields for research." Most of the work which has been done in these fields is inaccessible to many readers, so that such books as this one are not only necessary but are exceedingly wholesome.

The subject is divided into three parts. The first treats of the wave motions in matter or sound. The second, which occupies about half of the book, is devoted to electro-magnetic radiation or wave motions in the ether, ranging from the long electrical waves through the whole gamut to the short ultra-violet waves. In the last part of the book is given a clear and fairly complete treatment of those radiations which have not as yet been definitely classified as Cathode, Röntgen and Becquerel or Uranium rays. The book may be looked upon as a summary in a more or less popular manner of the work which has been done in recent years in these different kinds of radiation. The current theories in each case are discussed and, in general, are well handled. While the properties of the ether have been discussed rather fully, some parts of it have not received as clear a treatment as could be wished. This is especially true of Maxwell's theory. The matter condensed into pages 46 and 47 if expanded only a little more, would have increased the utility of the book.

It is to be regretted that the author found it necessary to introduce several new terms. As in general the introduction of new words into our scientific vocabulary is to be deprecated, there should exist a great demand to warrant the introduction of even one. The author has used *radiable* or *transradiable* instead of the word "transparent," and *nonradiable* instead of "opaque," the excuse being that the word "transparent," on account of its ordinary use, becomes ambiguous when used, for example, with reference to Röntgen rays. It is not clear how the mere change of a word would clear up the ambiguity. As evidence that the author himself

has not avoided it, one has only to read the paragraph about the middle of page 263. In using the words "transparent" and "opaque" it is only necessary to specify when using the words what kind of radiation one is referring to, as "transparent to Röntgen rays," or "opaque to ultra-violet rays," and this is necessary when using the new terms. The author has called electrical waves *infralacunal*, and the shorter waves, commonly called "light" waves, *ultralacunal*. As long as the gap or *lacuna* between the electrical waves and light waves exists there seems to be no urgent need for a change. When the gap is filled, if ever, then there may be a time for distinctions to be more clearly made and in that case the names suggested by the author will not suffice.

The great amount of work necessary in compiling a summary of such a character as this is seldom appreciated, and the reader of such a book should be prepared to overlook some errors. In this case there are not a great many, and these are not serious. One of the most misleading is the reference on pages 277 and 280 to the work done by Sagnac. From the author's statement it seems that Sagnac confirmed Becquerel's experiments on the reflection, refraction and polarization of Becquerel's rays. The article by Sagnac was not an account of his own experiments, but merely a summary of the work of Becquerel. So far as I have been able to learn no one has confirmed Becquerel's work on the reflection, refraction and polarization of these rays. The point is worth calling attention to as from more recent work (Rutherford, Phil. Mag., 47, p. 109, 1899, also Becquerel, Comptes Rendus, 128, p. 771, March, 1899), there exists doubt as to whether Becquerel rays possess these properties or not. This error may be due to the fact, that on account of the extent of field covered, the author depended on abstracts rather than on the original papers themselves. In the table on page 49, which is intended to show the entire range of ether vibrations, the author has omitted the work of Schumann (Abstracted in Beiblätter, p. 187, 1894), who measured ultra-violet wavelengths much shorter than any given in the table.

The work is well worth the perusal by students or teachers, or by any one desirous of getting better or clearer ideas of the various phenomena met in this very interesting field. As throughout the book references to the original sources have been given, it will serve as a useful bibliography to more advanced students.

O. M. STEWART.

NOTES.

American Association for the Advancement of Science.—The forty-eighth meeting of the Association was held at Columbus, Ohio, August 19–26, under the presidency of Dr. Edward Orton. The buildings of the Ohio State University afforded ample and unusually comfortable accommodation for the sectional meetings and the General Sessions. All things conduced to the uninterrupted work of the section, the excursions and entertainments being so arranged as not to interfere therewith, and the meetings of the council and executive committees being of moderate duration. That the General Session has outlived its years of usefulness was very evident; valuable time being spent in secretaries' announcements, which would be given sufficient publicity if inserted in the daily programme and posted on the bulletin board. The elimination or curtailment of these sessions is much desired.

Before Section B, in addition to the address by Vice-President Elihu Thomson, on "The Field of Experimental Research," the following forty-two papers were presented:

Apparatus for the demonstration of the varying currents in the different conductors of a rotary converter, F. C. Caldwell; A New Graphical Method of Constructing the Entropy-temperature Diagram from the Indicator Card of a Gas or Gil Engine, H. T. Eddy; Compound Harmonic Vibration of a String, W. Hallock; A New Form of Electrical Condenser having a Capacity Capable of Continuous Adjustment, L. J. Briggs; Time of Perception as a Measure of the Intensity of Light, J. McK. Cattell; Relations of Time and Space in Vision, J. McK. Cattell; The Musical Scales of the Arabs, C. K. Wead; Medieval Organ Pipes and their Bearing on the History of the Scale, C. K. Wead; Electrical Anesthesia, E. W. Scripture; An Absolute Determination of the E. M. F. of a Clark Cell, H. S. Carhart and K. E. Guthe; Quantitative Investigation of the Coherer, A. Trowbridge; Polarization and Polarization-Capacity, K. E. Guthe and M. D. Atkins; Current and Voltage Curves in the Magnetically Blown Arc and in the Aluminum Electrolytic Cell, R. A. Fessenden; Some New Apparatus—Tachometer, Chronograph, Data Collector, Induction Coil, Balance for Standardizing Ampèremeters, Standard of Induction, R. A. Fessenden; Measurement of Magnetism in Iron and the Relation between Permeability and Hysteresis, R. A. Fessenden; Polarization and Internal Resistance of the Copper Voltameter, B. E. Moore; Concerning the Fall of Potential at the Anode in a Geissler Tube, C. A.

Skinner; The Equipment and Facilities of the Office of U. S. Standard Weights and Measures for the Verification of Electrical Standard and Measuring Apparatus, F. A. Wolff, Jr.; An Experimental Test of the Accuracy of Ohm's Law, F. A. Wolff, Jr.; March Weather in the United States, etc., O. L. Fassig; A new Spectrophotometer and a Method of Optically Calibrating the Slit, D. B. Brace; On Achromatic Polarization in Crystalline Combinations, D. B. Brace; On the Nature of Electricity and Magnetism and a Determination of the Density and Elasticity of the Ether, R. A. Fessenden; Advances in Theoretical Meteorology, C. Abbe; on Differential Dispersion in Double Refracting Media, E. J. Rendtorff; Location of Smokeless Powder Discharge by Means of Colored Screens, R. A. Fessenden; a Method for the Study of Phosphorescent Sulphides, F. E. Kester; Accidental Double Refraction in Colloids and Crystalloids, B. V. Hill; Note on the Age of the Earth, R. A. Fessenden; a Bolometric Study of the Radiation of a Black Body Between 600° and 1100° C., C. E. Mendenhall; a Bolometric Study of the Radiation of an Absolute Black Body, F. A. Saunders; on Thermodynamic Surfaces of P. V. T. for Solid, Liquid and Gaseous State, F. E. Nipher; On the Escape of Gases from the Planets According to the Kinetic Theory, R. S. Cook; Relation of Magnetization to the Modulus of Elasticity, J. S. Stevens; On Flutings in Kundt's Tube, S. R. Cook; the Dielectric Strength of Oils, Thos. Gray; Some Unexpected Errors in Wattmeter Measurements, Thos. Gray; Note on the Preparation of Reticles, D. P. Todd; the Nature of Spoken Vowels, with Reference to the Theories of Helmholtz and Hermann, E. W. Scripture; Pressure and Wave-length, J. F. Mohler; the Attenuation of Sound and the Constant of Radiation, of Air, A. Wilmer Duff; Optical Calibration of the Slit of a Spectrometer, E. V. Capps.

The Association next meets at Columbia University, New York city, Monday, June 25th, under the presidency of R. S. Woodward. The officers of Section B elected for the New York meeting are: Vice-President, Ernest Merritt, Ithaca, N. Y.; Secretary, R. A. Fessenden, Allegheny, Pa.

An invitation was received to participate in the International Congress of Physicists to be held in Paris; a committee from Section B was appointed to acknowledge the invitation and to see that the Association is represented at the Congress.

International Congress of Physicists.—The French Physical Society has taken the initiative in arranging for an International Congress of Physicists to be held in connection with the Paris Exposition, from the 6th to the 12th of August, 1900. American physicists are invited to attend the Congress and to participate in its proceedings. Those who desire to be

present are requested to communicate with the secretaries: Ch. Ed. Guillaume, au Pavilon de Breteuil, Sèvres (Seine-et-Oise), and Lucien Poincaré, 105 bis, Boulevard Raspail, Paris.

ERRATUM.

Vol. IX., No. 3, Plate I. (Ramsey), opposite page 190.

For No. 2 read No. 33.
" " 4 " " 32.
" " 15 " " 11.
" " 11 " " 15.
" " 32 " " 4.
" " 33 " " 2.